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# Recalibrating reference within a dual-space interaction environment

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**Abstract** In this paper we examine how two groups of middle school students arrive at shared understandings of and solutions to mathematical problems. Our data consists of logs of student participation in the Virtual Math Teams (VMT) system as they work on math problems. The project supports interaction both through chat and through a virtual whiteboard. We have examined in detail, the sequential work these students do to constitute and specify 'the problem' on which they are working in the ways they produce whiteboard objects and text postings. Solutions emerge as students come to understand the problem on which they are working. This understanding is achieved through gradual respecification of the math problem on which they are working.

**Keywords** Indexicality · Referential practices · Problem solving · CSCL · Ethnomethodology

## Introduction

Collaborative math problem solving in online chat environments can be a tricky business. Students coming together in a CSCL environment to work on a math problem must figure out what problem they are working on and what they are referring to along the way as they communicate in the CSCL environment.

When students work together collaboratively in online environments, they tend to organize themselves to accomplish a shared understanding of the problem on which they are working. When a group of students is confronted with a problem-solving task, they must be able to (a) represent and refer to 'relevant' parts of the problem on which they are

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working and (b) know that everyone understands at least to some degree that which is being referenced (Parnafes 2010). The problem that students face is that they cannot know in advance of their engagement with problem resources and each other what counts as 'relevant' (Macbeth 2011). Much of the interactional work that students do in collaborative problem solving involves specifying the problem. This involves identifying, representing, referring to, and recognizing what become the problem's relevant constituent elements and properties in different semiotic modalities (online chat and whiteboard).

To get a sense of the issues that students face when working collaboratively on a math problem, consider that when a student attempts to convey to others a possible solution to the problem, all parties in the interaction must work out locally and in very practical terms how to actually refer to 'the problem' and how to formulate what 'the solution' might be. Referential practices, understood here as the ways that actors refer to and represent problems and solutions, are at the heart of the matter (Koschmann and Zemel 2009). In our view, students work to arrive at agreed-upon representations of relevant elements of the math problem on which they are working. The kinds of representations they use and the uses to which those representations are put constitute and constrain the trajectory of their engagement with and understanding of the problem on which they are working.

Any reference whose sense is dependent on the context of its use can be characterized as an indexical referent.[1] We align with ethnomethodological perspectives that hold that *all* references, expressions, accounts and the like are indexical (Garfinkel 1967; Garfinkel and Sacks 1970), that is, dependent on the circumstances of their occurrence for their local sense or meaning. We refer to the myriad ways in which they point or index into their context of production as their 'indexical properties'. Thus math problems are indexical phenomena that can be 'indexed' in various ways. Students constitute the problem on which they are working by indexing it, pointing to it, referring to its constituent properties, elements and features in particular ways. The more refined their referential work, viz., the more they recalibrate their referential practices to finer granularity of reference, the more refined their understanding of the problem.

In so-called 'dual-space' interaction environments, those that offer both chat interaction and sketching on a virtual whiteboard, it is necessary that others understand how the posting and the whiteboard object mutually constitute their sense and meaning (Cakir et al. 2009; Cakır 2009; Lonchamp 2009, 2011; Mühlpfordt 2006). Actors will routinely label, highlight, point to, or otherwise specify the object or matter of interest; but even then, confusions and misunderstandings may arise. Anticipating, encountering, and remedying these confusions involves an ongoing process of identifying, recognizing and reporting on properties of the object or matter in question as referentially relevant (Cakir et al. 2009; Mühlpfordt and Wessner 2009). If some object or matter is something students communicate about and work with, they must have a set of shared interactional resources that allow them to refer to that object or matter in mutually intelligible ways.

Representation has been a topic of considerable interest to scholars of mathematics and science education. Studies have examined representations produced and/or used by students engaged in problem-solving work in classroom contexts (Azevedo et al. 2012; Danish and Enyedy 2007; Danish and Phelps 2011; diSessa and Sherin 2000; diSessa 2004; Hall 1996; Medina and Suthers 2008; Parnafes 2010). In much of this literature, representations are routinely treated as artifacts with particular properties, the usefulness of which depends upon their design features. While the uses of representations are concerns that drive design

[1] "Indexicals are sometimes defined simply as expressions that change their reference from one context to the next" (Nunberg 1993, p. 2).



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considerations, representations themselves are treated as distinct from the referential uses to which they are put.

In CSCL settings, the issue of representation has been examined in terms of the design of external representational resources as well as the production and use of representations by students in terms of the affordances of the computer systems they use (Beers et al. 2005; Fischer and Mandl 2005; Kirschner and Van Bruggen 2004; Lonchamp 2011; Medina and Suthers 2008; Suthers et al. 2003; Suthers 2005; Van Bruggen et al. 2002; Van Bruggen and Kirschner 2003; White and Pea 2011). As with mathematics and science education, the approach frequently taken presumes that representations are phenomenal objects that, in their design and organization rather than their use, index cognitive and other phenomena (Vergnaud 1998). Roschelle's work on the Envisioning Machine (Rochelle and Teasley 1995; Rochelle 1996) examined how students work with external representations of physics concepts and the referential challenges they faced. Representational and referential concerns also arise when students try to produce, understand and share arguments or patterns of reasoning (Van Bruggen et al. 2002; van Drie et al. 2005).

In dual space, online CSCL systems, students routinely display their reasoning by sequentially producing objects on a whiteboard, or posting arguments in a chat area as though they are meaningfully connected by a set of shared reasoning practices (Cakir et al. 2009). It is often the case that students produce these sequences without much elaboration because they assume others can and will infer the reasoning steps involved from the sequential juxtaposition of relevant objects. To complicate matters, when students want to use the whiteboard to illustrate arguments they have made in the chat area, they are faced with the technical constraints of being able to work in only one area at a time (Mühlpfordt and Wessner 2009; Mühlpfordt 2006). This can make it difficult for some participants to know what object in the whiteboard is being referenced by some part of a chat posting.

Ethnomethodological studies of mathematical and scientific practice offer an alternative orientation (Garfinkel et al. 1981; Greiffenhagen and Sharrock 2005; Hester and Hester 2010; Koschmann and Zemel 2009; Livingston 1986; Lynch 1985, 1994, 2011; Psathas 2007; Schegloff 2000; Sharrock and Anderson 2011; Suchman 1988, 2006; Woolgar 1988). From this perspective, representations are materially manifest as particular kinds of referential practices by which actors come to specify and refer to the indexical properties of phenomena. We take an ethnomethodological perspective and see representation as a very particular form of 'objects-in-use'. Objects, be they drawings, gestures, graphs, texts, formulae, etc., are not themselves representations. We hold that representations are these objects *and* the way they are used in referential work. This makes representations referential resources used in the pursuit of interactional goals or outcomes that achieve their meaning through their referential use. In our view, no object is inherently representational in and of itself. It is only when that object is placed in the service of referring to the indexical properties of a phenomenon that it becomes a representation. According to Lynch (1994), "Wittgenstein and ethnomethodology inform us that the extent to which expressions and texts take on referential functions may owe less to the intrinsic properties of representational items than to the deeds performed when those items are embedded in action" (p. 5).

In this paper, we extend the research traditions of CSCL concerned with representation by treating it as a feature of referential practice. We examine how the indexical properties of underspecified objects emerge in the way that representations of these objects are sequentially accomplished and used in interaction. We look at how two groups of students build a problem by working out its referential properties. Specifically, we examine two cases in which a mathematical "problem" and its "solution" emerge as a recalibration of the referential properties of an emerging representation of the "problem." In doing so, students come to a shared, common understanding

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of these problems and their solutions. Students do so using the affordances of the system, resources in the problem statement sheet (an 'external representation' available on a wiki page) and a shared set of referential practices. It is up to the students themselves to identify in their reading of the problem statement's formulations and directives what the math problem is *for them* and then to work together to identify what *for them* might count as a solution to that problem. As we will see in the analysis that follows, this involves engaging in a process of calibrating and recalibrating reference to and representation of the relevant indexical properties of the problem and its solution in a way that achieves shared understanding.

## Virtual Math Teams

The materials to be discussed come from a corpus assembled at the Math Forum at Drexel University. The Virtual Math Teams (VMT) Project, established in 2003, is one of a variety of programs conducted under the auspices of the Math Forum. In this project, teams of geographically dispersed students use an integrated suite of web-based software tools to explore proposed mathematics topics (Stahl et al. 2006; Stahl 2009). VMT sessions are run as an enrichment activity conducted outside of the regular school curriculum. Students are recruited through their math teachers at their home schools.

The VMT environment is a multi-modal environment consisting primarily of a chat area and a shared whiteboard area (Mühlpfordt and Wessner 2005, 2009; Mühlpfordt 2006). Students interact by posting text to the chat area and by drawing figures or placing text in the whiteboard area (see Fig. 1). The system offers a number of affordances. Each participant is assigned a color in the chat area. Whiteboard actions appear in the whiteboard and are indexed by color-coded squares in the chat area that correspond to participants performing

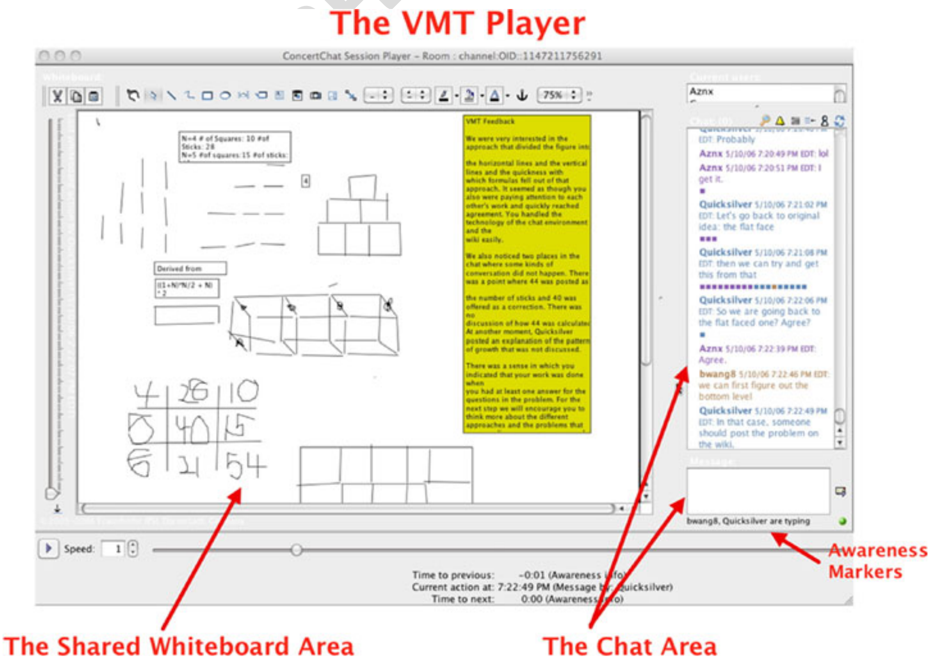


**Fig. 1** The VMT player

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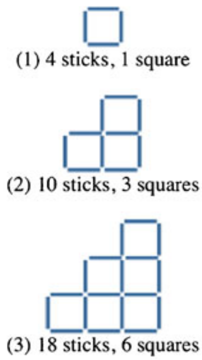
the whiteboard actions. Participants can “point” back to prior chat postings or to areas of the whiteboard using an indexing tool provided by the system.

The VMT system offers certain analytical tools as well. It captures all actions performed by actors or by the system in a log file that can be used in a “playback” technology that allows analysts to reproduce a display of the interactional environment of the VMT system. Postings and whiteboard actions can be played back in a way that shows exactly what an observer of the actual chat session would have seen as it occurred. This provides an analytical environment for investigating how the participants interacted using this online system.

In the spring of 2006, four teams of middle school students were recruited from around the country to participate in Springfest 2006. The team that displayed the best collaboration in approaching and coming up with a set of mathematical problems would receive an iPod as a prize. Members of the VMT staff monitored the teams, provided feedback, and assisted with technical questions about the VMT software. For the first session, the VMT staff provided resources on a wiki site for the teams to constitute for themselves a problem to solve (see Fig. 2).

**VMT Spring Fest**

Here are the first few examples of a particular pattern or sequence, which is made using sticks to form connected squares:




| N | Sticks | Squares |
|---|---|---|
| 1 | 4 | 1 |
| 2 | 10 | 3 |
| 3 | 18 | 6 |
| 4 | ? | ? |
| 5 | ? | ? |
| 6 | ? | ? |
| ... | ... | ... |
| N | ? | ? |

*Scroll down to see instructions for each Session.*

**Session I**

1. Draw the pattern for N=4, N=5, and N=6 in the whiteboard. Discuss as a group: How does the graphic pattern grow?
2. Fill in the cells of the table for sticks and squares in rows N=4, N=5, and N=6. Once you agree on these results, post them on the VMT Wiki
3. Can your group see a pattern of growth for the number of sticks and squares? When you are ready, post your ideas about the pattern of growth on the VMT Wiki.

**Fig. 2** The view topic wiki page

This "View Topic" wiki page makes available certain resources to the team for the purposes of beginning their participating in the VMT Springfest 2006. While one might be tempted to consider the contents of this page a "statement of the problem" for the students, it is only in the way that the assembled students orient to and organize their activities with respect to these resources that they come to discover what is for them the problem on which they will work. In short, through their reading practices and their online interaction, they work to constitute these resources into a problem to which they can give their attention and on which they can work.

It is also up to the students to produce a solution strategy, a solution and a report of that solution they are to post on a wiki. We selected two teams, Team B and Team C, to investigate. We examine how team members organize themselves and their analytical work to identify the problem they are working on and what might stand as a solution to that problem. We treat the work these students do as a form of "discovering work" (Garfinkel et al. 1981). This perhaps stretches the notion of discovery a bit, but we feel it is in keeping with the ordinary sense the participants have of (a) "figuring out" what the problem is and (b) "finding" the answer to that problem. For our students then, discovery involves a set of practices by which they orient to and acknowledge, as a discoverable phenomenon, a problem and a solution that are for them underspecified and are therefore subject to referential practices that allow the students as "members" (Garfinkel and Sacks 1970) to speak of, orient to, report on and account for the underspecified problem and its underspecified solution as discoverable matters.

It is well recognized that talk-in-interaction and text/graphics-in-interaction are different modalities of interaction that offer significantly different affordances for sense-making and communication by and among actors (Garcia and Jacobs 1999; Greiffenhagen 2008; Schönfeldt and Golato 2003; Zemel 2009). What we see in the VMT environment is that actors engaged in chat through VMT contend with the same set of concerns about referring to underspecified phenomena that actors in face-to-face interaction have. We also see that online chat affords different opportunities and resources for dealing with these concerns. For example, in the VMT environment, actors frequently use text-based referential terms in concert with white board demonstrations in ways that differ significantly from how a speaker might talk during and in relation to the construction of a graphical display in a face-to-face interaction. Unlike a mathematician presenting a proof at the blackboard (Greiffenhagen and Sharrock 2005), who can both talk and point at the same time, VMT participants can type in the chat window or draw on the whiteboard, but not both simultaneously because of practical constraints imposed by the computer interface. Thus online chat participants must organize the production of their textual and graphical postings in a serial sequence. In other words, a graphical demonstration can be constructed first followed by a text-based set of explanations, references, glosses, etc., or text-based explanations, references, glosses, etc., can be produced in the chat environment first which are then followed by graphical demonstrations on the whiteboard. They cannot happen simultaneously. Thus, online interactions can present participants with an interesting set of procedural concerns involving how reference and specification of relevant matters is achieved.

The online activities we examine are "reflexive, self-organizing, organized entirely in situ, locally" (Livingston 1987, p. 10). As such, they are available to an ethnomethodologically informed study. In practical terms, that means we are trying to understand the observable practices people perform from moment to moment to get things done in an organized, meaningful and accountable manner (Livingston 1987, 1999). In our examination of the VMT data, we investigate the students' mathematical reasoning as a set of accomplished referential and representational practices. The evidence suggests that, whatever else

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'arriving-at-an-understanding' might be, it is most intimately bound up with the emergent and shared use of a set of referential practices by which "experience, its retrieval in memory, and its shaping in discourse are designed by reference to context, co-participants, stance, the realization of action, and the trajectories of activity in which it is embedded" (Schegloff 2000, p. 718).

## The calibration of reference as problem-solving in VMT

In the Springfest '06 session data, actors rarely just presented completed solutions to the problems for ratification by others during their work sessions. The usual approach involved offering up displays and descriptions of their reasoning, grounded in the directives found in the resources found on the View Topics Page, as a way of eliciting recipient participation in the discovery of a problem's solution. This reasoning was often achieved as the sequential display of text postings and graphical objects that were designed to allow presenter and recipients to identify and reference more general mathematical representations of the specific examples or instances on which they were working.

In this analysis, we focus on two cases in which students use particular referential and representational practices to sequentially specify the problem on which they are working and its solution. Rather than distinguish between reference and representation as distinct phenomena, we see them as different aspects of a discovery and design process by which the relevant indexical properties of some underspecified object or matter are discovered and made available for referential use. As a practical matter, a representation becomes useful only when its indexical properties, the properties that allow for reference, are adequately specified and shared with other actors (Hanks 1992, 1996, 2000). We focus here on what Hanks (1990) termed *referential practices*. Thus, in the VMT system, when a student builds a 'representation' of a problem in a particular manner using some combination of text and graphics, it is not the 'representation' per se but the work of building the representation and working with it in a way that allows for the selection and identification of its relevant indexical properties that constitute the work of problem solving.

### Team B: "You can divide the thing into two parts"

In their first meeting, members of Team B (Bwang8, Aznx and Quicksilver) took up the problem of working out the pattern of growth in the number of sticks and squares in the figures shown on the View Topics page (see Fig. 2). After familiarizing themselves with the features of the VMT system, Bwang8 posts the following text: "you can divide the thing into two parts" (see Fig. 3, 6:32:05 PM; or Appendix 1, line 52).[2] This is a puzzle because two seemingly important elements of Bwang8's post are underspecified. There is no indication of (a) what "thing" refers to or (b) what "divide … into two parts" could mean. His statement, however, supplies an interpretative framing through which his subsequent actions on the whiteboard become meaningful. Bwang8's works on the white board immediately after posting his text, which suggests the possibility that his white board actions are to be seen as providing some kind of elaboration of the evidently-vague and underspecified set of mathematical actions proposed in his text posting.

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[2] A transcript of the text postings for Teams B and C are available in the Appendix 1 and 2, respectively.

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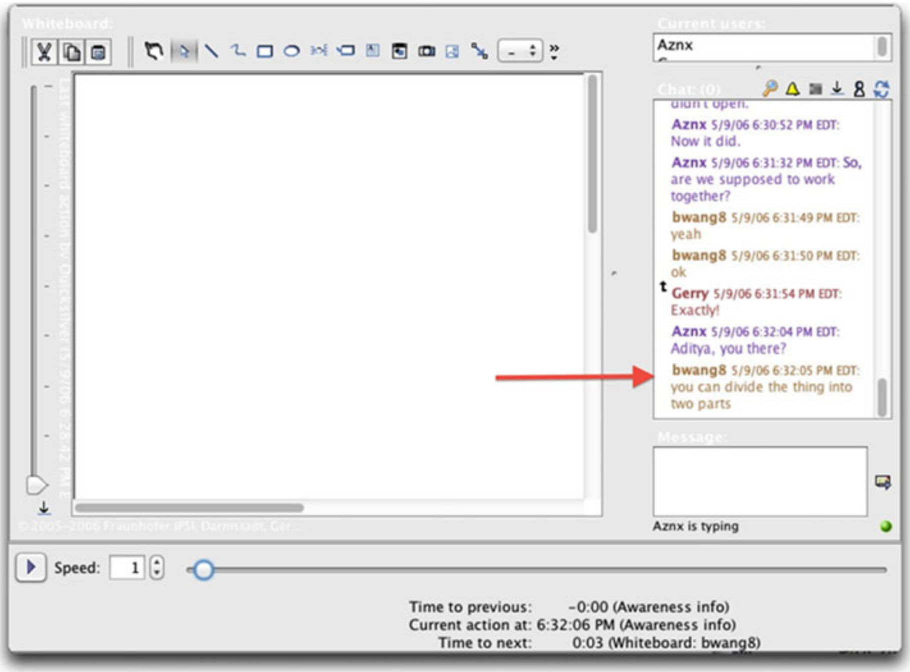


**Fig. 3** Bwang8's noticing

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Rather than identify in any explicit way what that "thing" is to which he refers, how it might be divided or even its relation to the problem, Bwang8 proceeds to systematically produce a set of unelaborated white board objects that, by virtue of being unelaborated, project the expectation that both Quicksilver and Aznx can or should be able identify "the thing" to which Bwang8 referred in his post.

The systematic manner with which Bwang8 produces the white board objects (see Fig. 4) suggests that he is producing a 'representation' of the "thing" to which he had referred. The representation is built line-by-line, as an emergent phenomenon. Bwang8's actions over time ultimately provide the indexical resources for identifying the object he is producing and demonstrates the nature of the division of that "thing" to which he has referred. The emergence of the "thing" through its systematic production on the whiteboard appears designed to make it possible for Aznx and Quicksilver to discover by witnessing Bwang8's blackboard work that the divisible object Bwang8 is producing corresponds to the $N$=3 stage object with 18 sticks and 6 squares shown on the View Topics page. What Bwang8 produces is not the same object, however. Bwang8's design work in producing these objects is systematically organized to make recognizable two mirror-image sets of lines, one horizontal and the other vertical, that in the order of their production and in their shape and distribution on the white board display how the $N$=3 stage figure viewable on the View Topics page can be divided into two parts. For Aznx and Quicksilver to recognize this object as the $N$=3 stage figure divided in two parts, they need to identify the relevant indexical properties of the graphical representation as related to the figure on the View Topics page.

However, as Bwang8 completes his whiteboard figures, Quicksilver, who had been resolving a problem with his computer, asks, "what are the lines for?" (see Fig. 5,

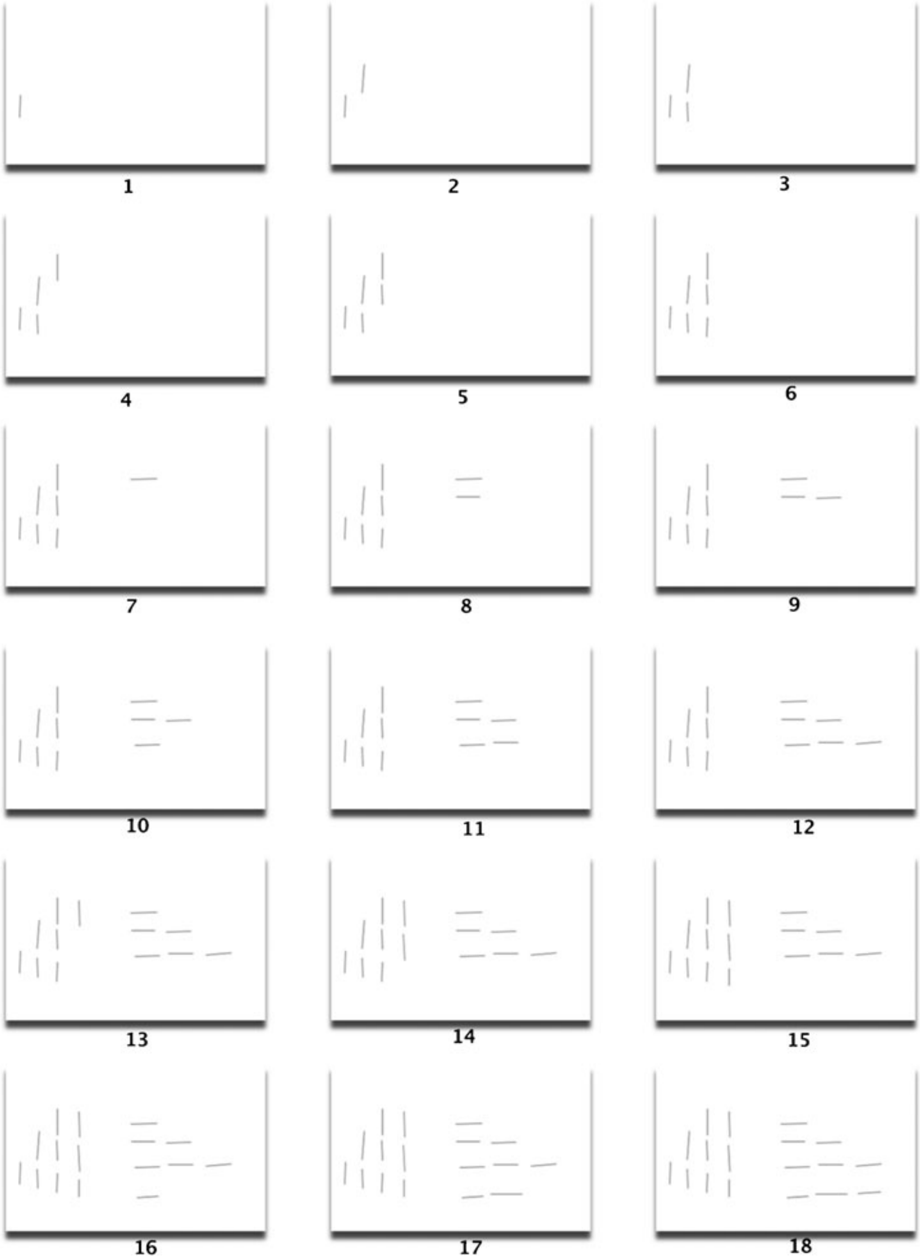


**Fig. 4** Production of Bwang8's "thing"

6:32:58 PM; or Appendix 1, line 56). Quicksilver's question suggests that he does not 'see' the connection between Bwang8's initial posting, his whiteboard actions and the problem on which they are working. Aznx advises Quicksilver to review the View Topics page to allow Quicksilver to see the figures shown there as representations in relation to Bwang8's graphical objects. Aznx's response to Quicksilver does not actually answer Quicksilver's

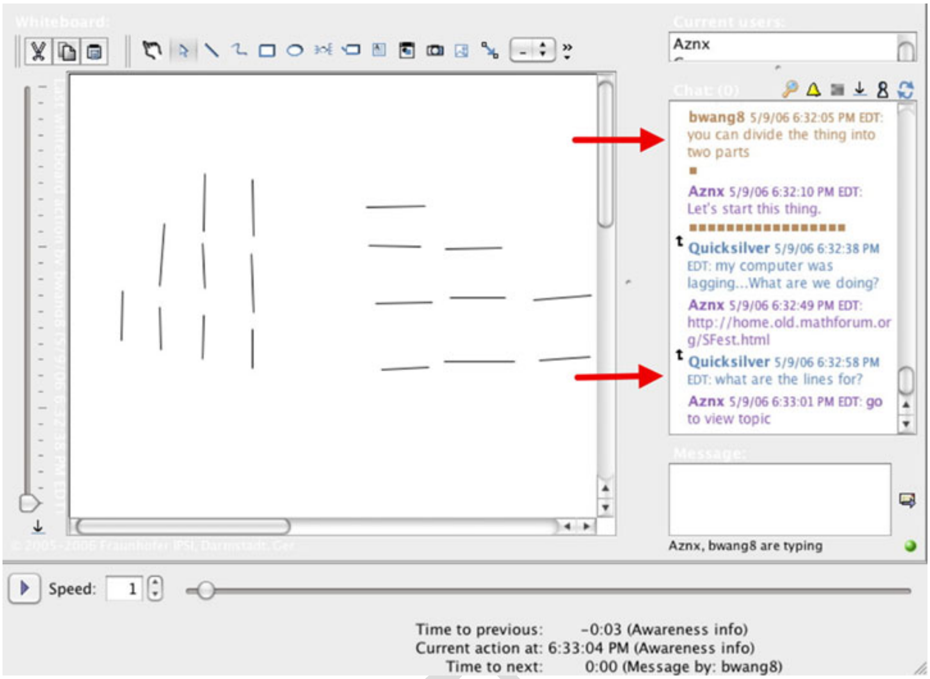


**Fig. 5** The "thing" divided in half

query but points him to the indexical resources necessary for recognizing what Bwang8's lines could be. This suggests that Aznx recognizes in Bwang8's referential and representational work a connection between the completed figure and the diagram on the View Topics page.

Upon completing his whiteboard objects, Bwang8 posts the following text in the chat area: "so you can see we only need to figure one out to get the total stick" (see Fig. 6, 6:33:05 PM; or Appendix 1, line 58). While not explicitly saying so, Bwang8 relies on Quicksilver and Aznx (a) to have witnessed the production and distribution of two symmetrical objects, one with horizontal lines and the other with vertical lines, and (b) to have treated the production of these objects and their symmetry as consequential for the mathematical formulation of a solution to the problem. In fact, Bwang8 treats the whiteboard object and the procedure of its production as the solution to the problem. All that remains is to 'represent' those objects and the procedures of their production into what is for them a suitable mathematical form. Bwang8 achieves this by respecifying the specific $N$=3 case as a more general algebraic formulation of the problem solution, "1+2+3+........+$N$+$N$" (see Fig. 6, 6:33:32 PM; or Appendix 1, line 60) and "times that by 2" (see Fig. 6, 6:33:38 PM; or Appendix 1, line 62).

Though Bwang8 has produced a mathematical formulation of the procedure he used to produce the whiteboard object, it was necessary to bring the other students to an understanding of his representation of the problem and its solution. Quicksilver had encountered technical difficulties as Bwang8 produced his whiteboard work, and had not necessarily followed the way the whiteboard objects had been produced. Aznx on the other hand appears to have been following Bwang8's work and responds to the initial formulation of the solution represented in algebraic form by recommending further algebraic simplification

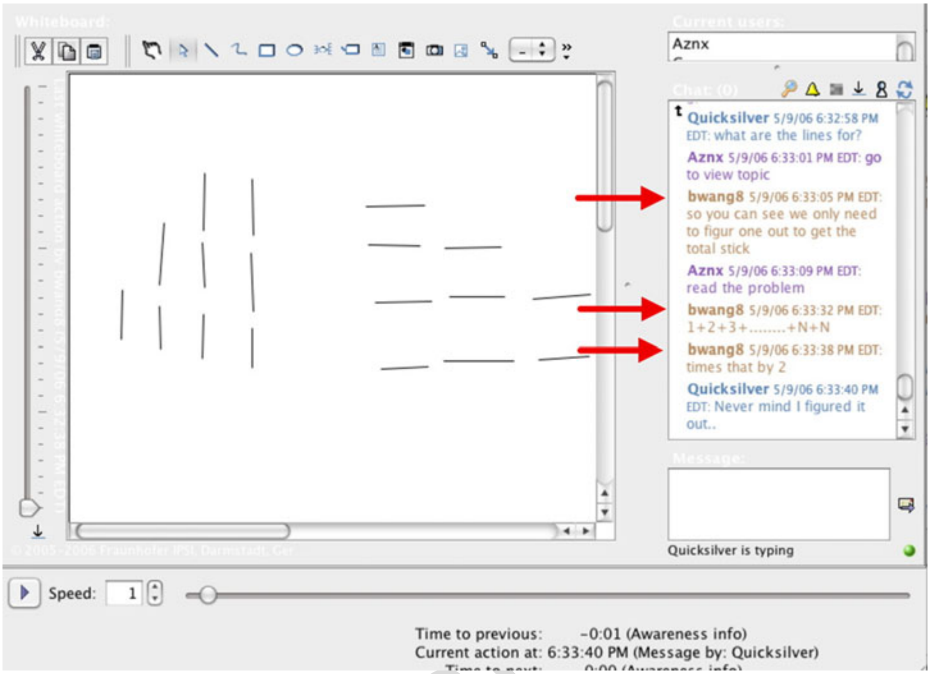


**Fig. 6** Bwang8 specifies a solution strategy

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(Appendix 1, lines 63, 64 and 66). Bwang8 accepts the recommendation and offers such a simplification, "((1+N)*N/2+N)*2" (Appendix 1, line 67), followed by a confirmation request (Appendix 1, line 68). Because the simplification was unelaborated, Bwang8 apparently presumed that the other participants would recognize his initial algebraic representation of his solution procedure (as worked out graphically on the whiteboard) as a Gaussian summation. Aznx calls for a derivation (Appendix 1, line 69). Bwang8 identifies his simplification as "a common formual (formula)" (Appendix 1, lines 71 and 72). What we see happening here is work between Bwang8 and Aznx to work out the relevant indexical properties of the proposed formulation, using the formulation itself to accomplish the referential work.

Taking stock, we see that Bwang8 began with an underspecified text version of a solution strategy. His first text posting, while built to be recognizable as a possible solution strategy, did not specify what was meant by such key terms as "thing" and "divide." In producing this post, Bwang 8 was making reference to what were then for him evidently vague properties of the problem and its solution. While it is clear that Quicksilver and Aznx did not immediately know what Bwang8 was referring to as "the thing," Bwang8's whiteboard work (Fig. 4) made it possible for him to demonstrate through its construction the "thing" divided in two parts. The enacted production of the divided object provided a basis for describing his procedure in a more precise and generalizable mathematical form, as "1+2+3+........+$N$+$N$" (see Fig. 6, 6:33:32 PM; or Appendix 1, line 60) times two, or subsequently in its reduced form as "((1+N)*N/2+N)*2" (Appendix 1, line 67). While Quicksilver struggles to 'catch up' with his partners, Aznx works to understand the references Bwang8 has

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made. Whether Bwang8 had the final version 'in mind' from the outset or 'discovered' it as he produced more specific mathematical versions of the solution based on the solution procedure he had enacted cannot be definitively established from the data. However the data clearly shows Bwang8 'recalibrating' his presentation by successively producing and introducing a richer set of indexical resources for specifying both the problem and its solution.

Team C: "Okay I've drawn $n$=4,5,6"

As the members of Team B were coming to terms with Bwang8's 'solution' to the problem, members of Team C, (Davidcyl, 137, Jason and Ssnish, were taking up the same problem but in a slightly different way. In Team C's case, the production of whiteboard objects preceded the posting of text in the chat area of the VMT system. Unlike Team B, where Bwang8's proleptic "You can divide the thing into two parts" creates a context for subsequent board work, Davidcyl begins without any chat posting and instead sequentially produces a figure on the whiteboard.

Davidcyl is the first team member to begin the session after the moderator's greeting. Rather than posting a text, or even responding to the moderator's greeting, Davidcyl produces a whiteboard object (see Fig. 7, note that the marker in the chat area indicates a whiteboard action has been performed). Davidcyl then proceeds to produce a series of squares and form them into first one object, then a second and finally a third, adding squares in a systematic manner to each successive object

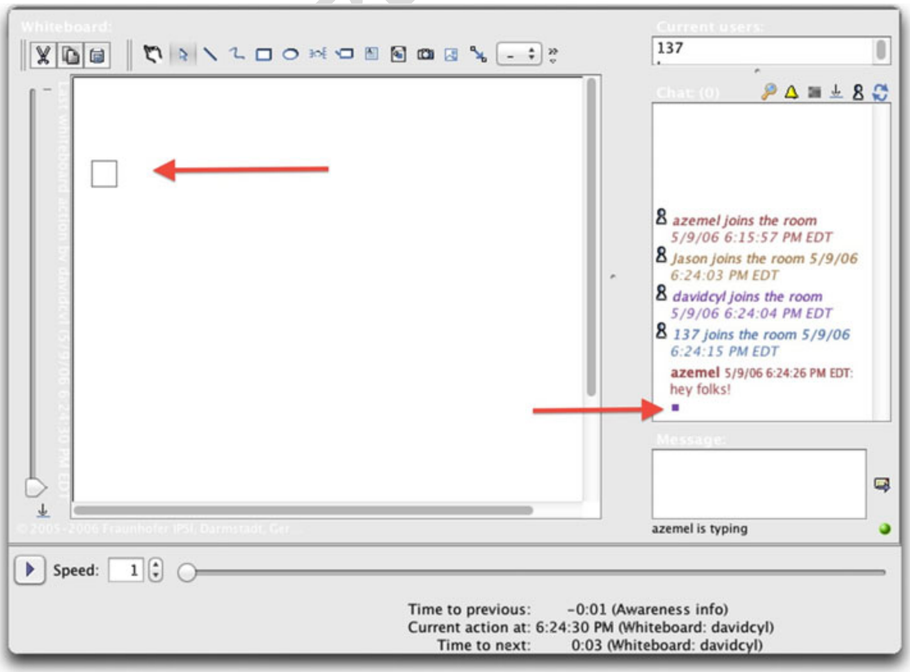


**Fig. 7** Davidcyl's first move

produced. The work consists of (1) building an initial object consisting of ten squares (steps 1 through 10), (2) duplicating that object (steps 11 and 12), (3) duplicating the squares from the longest edge and positioning them along that edge (steps 13 and 14), and (4) then adding a square to that edge (steps 15 and 16). A third object is constructed from the second object in a similar manner (steps 17 through 24). This is shown in Fig. 8.

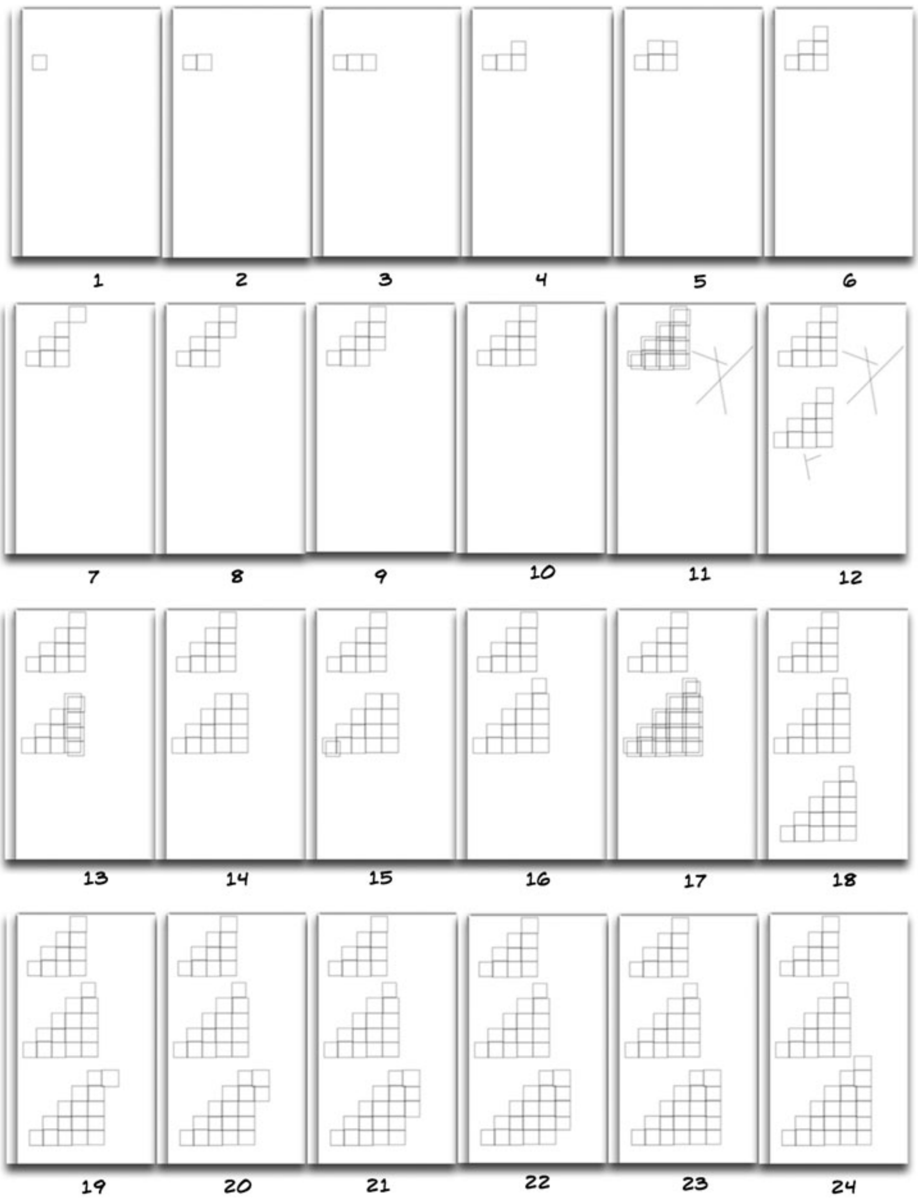


**Fig. 8** Performing the squares problem

Davidcyl's whiteboard work presents recipients with the problem of recognizing the relation between the sequentially unfolding design and production of representations on the whiteboard and some aspect of the problem presentation available on the View Topics page. Though members of Team C have access to the View Topic page with its diagrams and text, there is no explicit work done by any of the participants to indicate a link between Davidcyl's whiteboard work and the content of the View Topics page. In fact, as Davidcyl methodically produces his objects, 137 engages in some "doodling" on the whiteboard that, by taking no particular care with regard to Davidcyl's activity, treats Davidcyl's objects as a form of doodling as well (see panels 11 and 12 in Fig. 8). Davidcyl explicitly directs 137 to stop (Appendix 2, line 14), suggesting by this intervention that his whiteboard work is or will become consequential for their problem solving work. From that point forward, only Davidcyl works on the whiteboard until he announces the completion of his work, "okay I've drawn $n$=4,5,6" (see Fig. 9, 6:26:25 PM; or Appendix 2, line 22).

The intelligibility and significance of Davidcyl's whiteboard work has not been explicitly described prior to the sequential production of those objects. Participants are faced with the problem of sorting out the indexical properties of the objects Davidcyl has produced without any explicit indication of the significance of those objects. Jason started his own work in the text area while 137 began doodling (only to be stopped by Davidcyl). By declaring "okay i've drawn $n$=4,5,6," Davidcyl references resources from the View Topics page, resources with which other participants are presumed to be familiar, to indicate that (1) he has completed his whiteboard work

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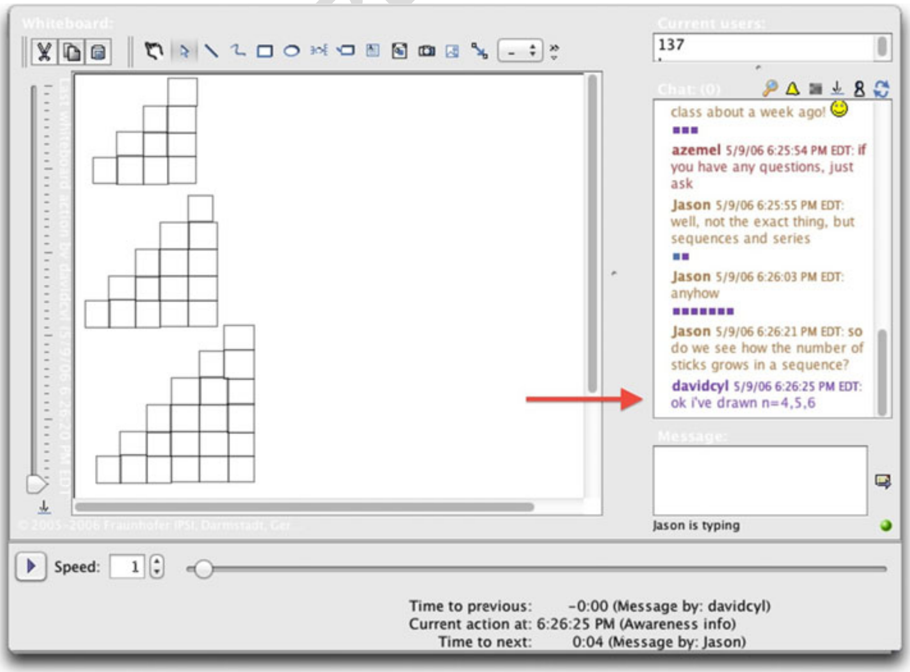


**Fig. 9** Davidcyl completes the objects

and (2) the three completed whiteboard objects are in response to the first problem task listed on the View Topics page, “Draw the pattern for $N=4$, $N=5$, and $N=6$ in the whiteboard” (see Fig. 2). By themselves and without Davidcyl’s post, “okay i’ve drawn $n=4,5,6$,” it is not obvious to the participants what the figures are, their possible or intended use, or their relationship to the problem. By producing a text posting upon completing the figures, Davidcyl is offering an account of these objects that indexes the shared problem resources available to them.

As it turns out, it is not simply the three figures that matter for their work. The way these figures were produced is consequential for the formulation of the mathematical reasoning that will turn out to be the solution to the problem. However, before Davidcyl can produce this formulation, Jason offers up text postings with regard to the issue of the sequential growth of the number of squares. Simply identifying the figures as “$n=4,5,6$” does not appear to reveal the sense of the objects to the other recipients, who continue to pursue other ways of specifying the problem (Appendix 2, lines 23, 24 and 25).

As Jason presents his own reasoning with regard to their task (see Fig. 10), Davidcyl rearranges the figures on the whiteboard. He appears to take no notice of Jason’s immediately subsequent text postings and instead produces his own post in which he reports, “the nth pattern has $n$ more squares than the (n-1)th pattern” (See Fig. 10, 6:27:32; or Appendix 2, line 26). Davidcyl then formulates this description in more mathematically specific representation in the next post: “basically it’s $1+2+..+(n-1)+n$ for the number of squares in the nth pattern” (Appendix 2, line 27). An

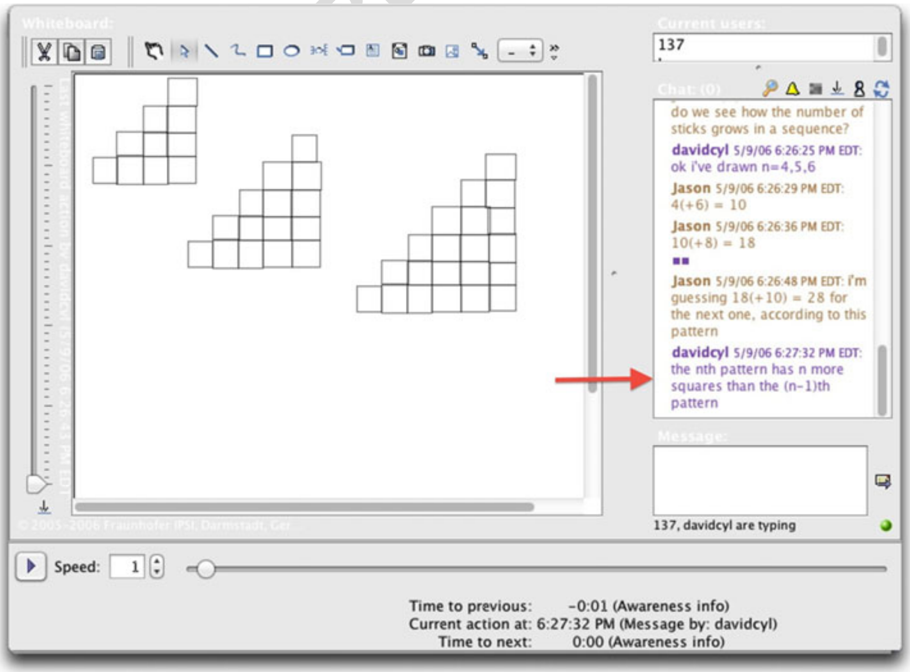


**Fig. 10** Davidcyl formulates the sequence

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algebraic variable is the ultimate indexical, one that holds its object of reference as open. Here, the variable $n$ achieves its denotational sense, both in terms of Davidcyl's use of "$n$=4,5,6," as a descriptor for what he has drawn and because of the use of "N" in the original task description. In response to Davidcyl's mathematicized formulation of the procedure by which the whiteboard objects were constituted, 137 responds with "so $n(n+1)/2$" (Appendix 2, line 28), extending Davidcyl's work to give the simplified version of the Gaussian sum. As 137 is composing his response to Davidcyl, Davidcyl is preparing a follow up: "and we an use the Gaussian sum to determine the sume: $n(1+n)/2$" (Appendix 2, line 29). It is at this point that both 137 and Davidcyl have achieved a shared understanding of the problem and its solution. Davidcyl remarks, "137 got it" (Appendix 2, line 30).

### Specifying the sense of objects and texts as a sequential achievement

The work Davidcyl did in Team C contrasts with and complements the work Bwang8 did in Team B. In both cases, participants relied on and utilized the contents of the View Topics page as referential resources in producing and interpreting the respective representations of problem solutions. The sense of these resources, however, emerges only through the situated, local and sequential production of the whiteboard objects and text postings. In the first case, Bwang8 produced an underspecified text posting whose sense was elaborated in the procedure he used subsequently to constitute the whiteboard objects that demonstrated how to understand the underspecified indexical terms "thing" and "divide … in half". Davidcyl, on the other hand, produced objects and a procedure for their production to which he could make reference in his subsequent text postings ("$n$=4,5,6," "pattern" and "squares"). The semiotic resources of the View Topics page, the whiteboard representations produced and the procedure of their production on the whiteboard, and the text postings with their indexical terms provided both teams with the resources required to discover the problem being solved and its solution.

Because one cannot produce texts that simultaneously narrate the production of whiteboard objects in the VMT environment, how actors accomplish the production of texts and objects is consequential for their intelligibility. In both cases we examined, the actors exploited this as an affordance of the VMT system, making it possible for both the objects produced and the sequential organization of their production to contribute to the specific sense of prior text posting in Team B's case and subsequent text postings in Team C's case. Thus it was the way that Bwang8 produced the line objects that made evident what he was referring to in his post when he wrote, "you can divide the thing into two parts." Likewise, it was the way that Davidcyl constructed each successive 'pyramid' as a copy of a prior figure to which were added an increasing but specific number of additional squares in a particular way, that provided for his formulation of the sequence in mathematical terms, "the $n$th pattern has $n$ more squares than the ($n$−1)th pattern." While we only examined these two cases, this suggests that the emergent recalibration and specification of the indexical properties of the problem constitute the conceptual achievement of the students.

## Recalibration of reference as discovering work in VMT

In a landmark ethnomethodological study of how people do mathematical proof, Eric Livingston (2000) wrote:

> "[P]rovers while engaged in the work of proving, are hunting for proofs in that work. The conception of proofs and their associated theorems as discrete

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> entities, somehow separated from provers continual, omnipresent engagement in the activity of proving, is simply false. *From within proving's work, the prover seeks to 'pair' a written description with a discovered, and often quite nonlinear, gestalt of reasoning*." (p. 265, emphasis added)

We too are interested in the lived work of mathematics as a kind of discovery. Like Livingston, we are interested in the local and situated practices and procedures by which actors refine and specify the indexical properties of an emergent math problem on which they are working. We are not dealing with mathematicians well-versed and trained in the referential and representation practices of professional mathematics, but middle school students engaged in mathematical problem solving. These students, like their professional counterparts, are engaged in the lived work of understanding the problems on which they are working. In our approach, we recognize that understanding is accomplished and shared in the referential practices by which actors constitute their emerging representations of the problems they face. We build on earlier research (Koschmann and Zemel 2009, 2011) that explores the nature of discovery in broader terms. In that earlier work, we argued that "discovering work" (Garfinkel et al. 1981) consists of the work that actors do to specify relevant properties of some noticed matter, viz. its indexical properties, in a way that allows others to refer to those properties as well. In other words, discovering work involves recalibrating referential practices that consists of the pursuit and production of greater referential specificity or granularity (Schegloff 2000) using natural language and/or other semiotic resources with regard to an underspecified "object-of-sorts with neither demonstrable sense nor reference" (Garfinkel et al. 1981, p. 135). The concerted work of this kind of specification is part of how underspecified but noticeable features of the phenomenal world, including math problems and their solutions, become known and available as social facts.

So, how can the work of Bwang8 and Davidcyl be seen as discovering work rather than just the production of a demonstration of an already-achieved solution to the problem? We cannot know definitively, but these two cases suggest that by looking at referential practices, we might be able to point towards an answer. Had this been a demonstration of an already-achieved solution, the indexical properties of the problem would have already been worked out by the presenters, Davidcyl and Bwang8, and made available to the other participants. However, for both Bwang8 and Davidcyl, the referential and representational resources and practices they came to use emerged as constituent features of their presentations. Neither Bwang8 nor Davidcyl presented a set of terms, features, assumptions or any other such mathematical specifications (other than those resources available in the problem statement) prior to the demonstrations by which recipients might have been able to follow the reasoning in either the text-based work or the whiteboard work. No explicit explanations of objects, terms, or reasoning were produced as they might be when instructing others to recognize an already-worked-out solution. Instead, it seems that they were working out these matters as they came to recognize them during the sequential production of their whiteboard objects and text postings. The specific indexical or referential properties of the problem emerge in the way the whiteboard objects and text postings were sequentially produced in relation to each other. The work done by Davidcyl and Bwang8 involves producing greater specification of texts and objects in the VMT system through the sequential

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production of objects and texts as mutually referential and constitutive domains of interaction.

The emergent nature of the indexical properties of the text and objects these students work on/with is highlighted by the fact that the VMT environment, by virtue of its technical affordances, does not allow participants to 'narrate' the production of whiteboard figures as one might do while drawing on a chalkboard in a classroom full of students. Instead, VMT participants are constrained to (1) produce a text posting in advance of their whiteboard work that prepares recipients for the whiteboard work that is to be done, or (2) produce whiteboard figures first and then account for the production of these figures after they have been produced. This means that whenever actors seek to mutually constitute the sense of texts and objects, recipients either must await the appearance of the object after the production of a text and then discover how the indexical properties of the text map onto the whiteboard object or they must await the appearance of a text after the production of a whiteboard object and discover how the indexical properties of that object map onto the text. In short, greater specification of texts and objects in the VMT system is achieved in the sequential production of objects and texts as mutually referential domains of interaction. Actors exploit the design features of any system to identify what they can accomplish with that system (Hutchby 2001). Not only are design recommendations difficult to identify as a result, they are beyond the scope of this paper.

The emphasis in this paper has been on the ways that individual actors produce greater referential specificity regarding the indexical properties of the mathematical problems on which they are working. We saw that recipients of these recalibrating references oriented to or took up these recalibrations in their recognition of when to examine the problem statements page on the VMT wiki and in their assessment of the various proffered solution formulations. While it remains to be seen what happens when the work of recalibration is interactionally problematic, these examples show that the achievement of the problem, not just the solution, arises from the ways that actors calibrate their representations of and references to problem elements in increasingly specific ways.

This leads us to the broader question we consider in this paper: How do interlocutors refer to and represent unknown, underspecified or poorly understood matters? It is clearly the case that people routinely do refer to, discuss, represent, and specify matters that they do not fully understand, that are unknown to them or that are in relevant ways underspecified.[3] In order to engage in interaction with regard to such underspecified matters, actors attempt to specify noticed features of that underspecified matter, viz. its indexical properties, in ways that allow others to refer to that object and its indexical properties. In such circumstances, the distinction between representation and referential practice effectively collapses. In order to refer to an underspecified matter, actors must identify or discover its indexical properties, properties that through their specification, articulation and formulation become, for the purposes at hand, both a representation of the matter and the means by which it is referenced (cf. Koschmann and Zemel 2009; Zemel et al. 2008). It is precisely this

[3] Garfinkel et al. refer to these as "object[s]-of-sorts with neither demonstrable sense nor reference" (1981, p. 135).

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work of specifying the indexical properties of unknown things that allows what was previously unknown to become known.

## Appendix 1

t1.1 **Table 1** Team B Chat log

| | Chat Index | Time of Posting | Author | Content |
|---|---|---|---|---|
| t1.2 | Chat Index | Time of Posting | Author | Content |
| t1.3 | 52 | 06.32.05 PM | bwang8 | you can divide the thing into two parts |
| t1.4 | 53 | 06.32.10 PM | Aznx | Let's start this thing. |
| t1.5 | 54 | 06.32.38 PM | Quicksilver | my computer was lagging…What are we doing? |
| t1.6 | 55 | 06.32.49 PM | Aznx | http://home.old.mathforum.org/SFest.html |
| t1.7 | 56 | 06.32.58 PM | Quicksilver | what are the lines for? |
| t1.8 | 57 | 06.33.01 PM | Aznx | go to view topic |
| t1.9 | 58 | 06.33.05 PM | bwang8 | so you can see we only need to figur one out to get the total stick |
| t1.10 | 59 | 06.33.09 PM | Aznx | read the problem |
| t1.11 | 60 | 06.33.32 PM | bwang8 | 1+2+3+........+$N$+$N$ |
| t1.12 | 61 | 06.33.38 PM | bwang8 | times that by 2 |
| t1.13 | 62 | 06.33.40 PM | Quicksilver | Never mind I figured it out.. |
| t1.14 | 63 | 06.34.01 PM | Aznx | Can we collaborate this answer even more? |
| t1.15 | 64 | 06.34.05 PM | Aznx | To make it even simpler? |
| t1.16 | 65 | 06.34.15 PM | bwang8 | ok |
| t1.17 | 66 | 06.34.16 PM | Aznx | Because I think we can. |
| t1.18 | 67 | 06.34.50 PM | bwang8 | ((1+N)*N/2+N)*2 |
| t1.19 | 68 | 06.34.58 PM | bwang8 | that's the formula, right? |
| t1.20 | 69 | 06.35.15 PM | Aznx | How did you come up with it? |
| t1.21 | 70 | 06.35.16 PM | bwang8 | for total sticks |
| t1.22 | 71 | 06.35.34 PM | bwang8 | is a common formual |
| t1.23 | 72 | 06.35.40 PM | bwang8 | formula |
| t1.24 | 73 | 06.35.46 PM | Aznx | Yeah, I know. |
| t1.25 | 74 | 06.35.59 PM | bwang8 | and just slightly modify it to get this |
| t1.26 | 75 | 06.36.31 PM | Aznx | Aditya, you get this right? |
| t1.27 | 76 | 06.37.45 PM | Quicksilver | What does the $n$ represent? |
| t1.28 | 77 | 06.37.57 PM | bwang8 | the given |
| t1.29 | 78 | 06.37.58 PM | bwang8 | N |
| t1.30 | 79 | 06.38.02 PM | Aznx | Yeah. |
| t1.31 | 80 | 06.38.05 PM | Aznx | In the problem. |
| t1.32 | 81 | 06.38.37 PM | Quicksilver | Oh |
| t1.33 | 82 | 06.38.38 PM | bwang8 | The number of squares is just (1+N)*N/2 |
| t1.34 | 83 | 06.38.50 PM | Quicksilver | We need that as well. |

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## Appendix 2

**Table 2** Team C Chat log

| Chat index | Time of posting | Author | Content |
|---|---|---|---|
| 1 | 06:15:57 PM | azemel | joins the room |
| 2 | 06:24:03 PM | Jason | joins the room |
| 3 | 06:24:04 PM | davidcyl | joins the room |
| 4 | 06:24:15 PM | 137 | joins the room |
| 5 | 06:24:26 PM | azemel | hey folks! |
| 6 | 06:24:39 PM | azemel | is everyone here? |
| 7 | 06:24:42 PM | Jason | |
| 8 | 06:24:44 PM | davidcyl | yeaqh |
| 9 | 06:24:45 PM | 137 | YA. |
| 10 | 06:24:46 PM | Jason | 4 people |
| 11 | 06:24:52 PM | azemel | great! |
| 12 | 06:25:03 PM | azemel | be sure to click on the view topic button |
| 13 | 06:25:18 PM | azemel | up at the top of the vmt screen |
| 14 | 06:25:22 PM | davidcyl | 137 stop |
| 15 | 06:25:29 PM | 137 | Oops. |
| 16 | 06:25:37 PM | davidcyl | np |
| 17 | 06:25:44 PM | Jason | ooh we just did this in math class about a week ago! :-) |
| 18 | 06:25:54 PM | azemel | if you have any questions, just ask |
| 19 | 06:25:55 PM | Jason | well, not the exact thing, but sequences and series |
| 20 | 06:26:03 PM | Jason | anyhow |
| 21 | 06:26:21 PM | Jason | so do we see how the number of sticks grows in a sequence? |
| 22 | 06:26:25 PM | davidcyl | ok i've drawn $n$=4,5,6 |
| 23 | 06:26:29 PM | Jason | 4(+6)=10 |
| 24 | 06:26:36 PM | Jason | 10(+8)=18 |
| 25 | 06:26:48 PM | Jason | i'm guessing 18(+10)=28 for the next one, according to this pattern |
| 26 | 06:27:32 PM | davidcyl | the nth pattern has $n$ more squares than the $(n-1)$th pattern |
| 27 | 06:27:55 PM | davidcyl | basically it's $1+2+..+(n-1)+n$ for the number of squares in the nth pattern |
| 28 | 06:28:16 PM | 137 | so $n(n+1)/2$ |
| 29 | 06:28:24 PM | davidcyl | and we can use the gaussian sum to determine the sum: $n(1+n)/2$ |
| 30 | 06:28:36 PM | davidcyl | 137 got it |

## References


Azevedo, F. S., diSessa, A. A., & Sherin, B. L. (2012). An evolving framework for describing student engagement in classroom activities. *The Journal of Mathematical Behavior, 31*(2), 270–289. doi:10.1016/j.jmathb.2011.12.003.

Beers, P. J., Boshuizen, H., Kirschner, P. A., & Gijselaers, W. H. (2005). Computer support for knowledge construction in collaborative learning environments. *Computers in Human Behavior, 21*(4), 623–643.


UNCORRECTED PROOF

Retrieved from http://igitur-archive.library.uu.nl/fss/2006-0920-200204/kirschner_05_computer_support_knowledge_construction_collaborative.pdf.

Cakır, M. P. (2009). *How online small groups co-construct mathematical artifacts to do collaborative problem solving.* Drexel University. Q2

Cakir, M. P., Zemel, A., & Stahl, G. (2009). The joint organization of interaction within a multimodal CSCL medium. *International Journal of Computer-Supported Collaborative Learning, 4*(2), 115–149.

Danish, J. A., & Enyedy, N. (2007). Negotiated representational mediators: How young children decide what to include in their science representations. *Sci. Ed., 91*(1), 1–35. doi:10.1002/(ISSN)1098-237X.

Danish, J. A., & Phelps, D. (2011). Representational practices by the numbers: How kindergarten and first? grade students create, evaluate, and modify their science representations. *International Journal of Science Education, 33*(15), 2069–2094. doi:10.1080/09500693.2010.525798.

diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and Instruction, 22*(3), 293–331. doi:10.1207/s1532690xci2203_2.

diSessa, A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *The Journal of Mathematical Behavior, 19*(4), 385–398. Retrieved from http://www.sciencedirect.com/science/article/pii/S0732312301000517.

Fischer, F., & Mandl, H. (2005). Knowledge convergence in computer-supported collaborative learning: The role of external representation tools. *The Journal of the Learning Sciences, 14*(3), 405–441. Retrieved from http://telearn.archives-ouvertes.fr/docs/00/70/40/01/PDF/Fischer_Mandl_2005.pdf.

Garcia, A., & Jacobs, J. B. (1999). The eyes of the beholder: Understanding the turn-taking system in quasi-synchronous computer-mediated communication. *Research on Language and Social Interaction, 32*(4), 337–367.

Garfinkel, H. (1967). *Studies in ethnomethodology.* Cambridge: Polity Press.

Garfinkel, H., & Sacks, H. (1970). On formal structures of practical actions. *Theoretical sociology: Perspectives and developments,* 337–366. Q3

Garfinkel, H., Lynch, M., & Livingston, E. (1981). The work of a discovering science constructed with materials from the optically discovered pulsar. *Philosophy of the Social Sciences, 11*(2), 131–158.

Greiffenhagen, C. (2008). Unpacking tasks: The fusion of new technology with instructional work. *Computer Supported Cooperative Work (CSCW), 17*(1), 35–62. Retrieved from http://www.springerlink.com/index/N0760P8P725N2266.pdf.

Greiffenhagen, C., & Sharrock, W. (2005). Gestures in the blackboard work of mathematics instruction. *Paper presented at the 2nd Conference of the International Society for Gesture Studies (Interacting Bodies), Lyon France*. Retrieved from http://gesture-lyon2005.ens-lsh.fr/IMG/pdf/Greiffenhagen-Gesture.pdf

Hall, R. (1996). Representation as shared activity: Situated cognition and Dewey's cartography of experience. *Journal of the Learning Sciences, 5*(3), 209–238.

Hanks, W. F. (1990). *Referential practice: Language and lived space among the Maya*. Chicago: University of Chicago Press.

Hanks, W. F. (1992). The indexical ground of deictic reference. In A. Duranti & C. Goodwin (Eds.), *Rethinking context: Language as an interactive phenomenon* (Vol. 11, pp. 43–76). Cambridge: Cambridge University Press.

Hanks, W. F. (1996). *Language and communicative practices*. Boulder: Westview.

Hanks, W. F. (2000). *Intertexts: Writings on language, utterance, and context*. Lanham: Rowman & Littlefield.

Hester, S., & Hester, S. (2010). Conversational actions and category relations: An analysis of a children's argument. *Discourse Studies, 12*(1), 33–48. doi:10.1177/1461445609347233.

Hutchby, I. (2001). *Conversation and technology: From the telephone to the internet*. Cambridge: Polity Press.

Kirschner, P. A., & Van Bruggen, J. (2004). Learning and understanding in virtual teams. *CyberPsychology & Behavior, 7*(2), 135–139. Retrieved from 10.1089/109493104323024401 http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=12934823&site=ehost-live.

Koschmann, T., & Zemel, A. (2009). Optical pulsars and black arrows: discoveries as occasioned productions. *Journal of the Learning Sciences, 18*(2), 200–246.

Koschmann, T., & Zemel, A. (2011). "So that's the ureter". the informal logic of discovery work. *Ethnographic Studies, 12*, 31–46.

Livingston, E. (1986). *The ethnomethodological foundations of mathematics*. London: Routledge & Kegan Paul.

Livingston, E. (1987). *Making sense of ethnomethodology.* London: Routledge & Kegan Paul.

Livingston, E. (1999). Cultures of proving. *Social Studies of Science, 29*(6), 867–888.

Livingston, E. (2000). The availability of mathematics as an inspectable domain of practice through the use of origami. In S. Hester & D. Francis (Eds.), *Local education order* (pp. 245–270). Amsterdam: John Benjamins.

UNCORRECTED PROOF

EDITOR'S PROOF JrnlID 11412_ArtID 9164_Proof# 1 - 08/01/2013




Lonchamp, J. (2009). A three-level analysis of collaborative learning in dual-interaction spaces. *International Journal of Computer-Supported Collaborative Learning, 4*(3), 289–317. Retrieved from http://www.springerlink.com/index/c8344h6738426463.pdf.

Lonchamp, J. (2011). *Deixis in synchronous CSCL systems*. Proceedings from 3rd Conference on Computer Supported Education - CSEDU 2011.

Lynch, M. (1985). Discipline and the material form of images: An analysis of scientific visibility. *Social Studies of Science, 15*(1), 37.

Lynch, M. (1994). Representation is overrated: Some critical remarks about the use of the concept of representation in science studies. *Configurations, 2*(1), 137–149. Retrieved from http://www.petajwhite.net/Uni/910/Legit and Representation/Representation Precis/Lynch.doc.

Lynch, M. (2011). Credibility, evidence, and discovery: The case of the ivory-billed woodpecker. *Ethnographic Studies, 12*, 78–105.

Macbeth, D. (2011). Understanding understanding as an instructional matter. *Journal of Pragmatics, 43*(2), 438–451. doi:10.1016/j.pragma.2008.12.006.

Medina, R., & Suthers, D. D. (2008). *Bringing representational practice from log to light.*

Mühlpfordt, M. (2006). Dual interaction Spaces: Integration Synchroner Kommunikation und Kooperation. *4. e-Learning Fachtagung Informatik*.

Mühlpfordt, M., & Wessner, M. (2005). Explicit referencing in chat supports collaborative learning. In T. Koschmann, D. D. Suthers, & T.-W. Chan (Eds.), *Computer supported collaborative learning: The next 10 years!* (pp. 460–469). Mahwah: Erlbaum.

Mühlpfordt, M., & Wessner, M. (2009). The integration of dual-interaction spaces. *Studying virtual math teams,* 281—293. Retrieved from http://gerrystahl.net/vmt/book/15.pdf

Nunberg, G. (1993). Indexicality and deixis. *Linguistics and Philosophy, 16*(1–2), 1–43.

Parnafes, O. (2010). *Representational practices in the activity of student-generated representations (SGR) for promoting conceptual understanding.*

Psathas, G. (2007). Lebenswelt origins of the sciences. *Hum Stud, 30*(1), 1–2. doi:10.1007/s10746-007-9047-8.

Rochelle, J. (1996). Designing for cognitive communication: Epistemic fidelity or mediating collaborative inquiry. In D. L. Day & D. K. Kovacs (Eds.), *Computers, communication and mental models* (pp. 13–25). London: Taylor & Francis.

Rochelle, J., & Teasley, S. D. (1995). The construction of shared knowledge in collaborative problem solving. In C. O'Malley (Ed.), *Computer-supported collaborative learning, NATO ASO Series F: Computer and system sciences (Vol. 128)* (pp. 69–97). Berlin: Springer.

Schegloff, E. (2000). On granularity. *Annual Review of Sociology, 26*, 715–720.

Schönfeldt, J., & Golato, A. (2003). Repair in chats: A conversation analytic approach. *Research on Language and Social Interaction, 36*(3), 241–284.

Sharrock, W., & Anderson, B. (2011). Discovering a practical impossibility: The internal configuration of a problem in mathematical reasoning. *Ethnographic Studies, 12*, 47–58.

Stahl, G. (2009). *Studying virtual math teams*. New York: Springer.

Stahl, G., Zhou, N., & Toledo, R. (2006). The Virtual Math Teams project: A global math discourse community. *International Conference on Computers and Education (ICCE '06)*. Retrieved from http://www.cis.drexel.edu/faculty/gerry/pub/icce2006.pdf

Suchman, L. A. (1988). Representing practice in cognitive science. *Human Studies, 11*(2), 305–325.

Suchman, L. A. (2006). *Human–machine reconfigurations: Plans and situated actions (learning in doing: Social, cognitive and computational perspectives)* (2nd ed.). Cambridge: Cambridge University Press.

Suthers, D. D. (2005). Collaborative knowledge construction through shared representation. *38th Hawai'i International Conference on the System Sciences*,.

Suthers, D., Girardeau, L., & Hundhausen, C. (2003). Deictic roles of external representations in face-to-face and online collaboration. *Wasson, Ludvigsen & Hoppe (2003), op. cit,* 173—182. Retrieved from http://metheny.ics.hawaii.edu/papers/2003/Suthers-et-al-CSCL2003.pdf

Van Bruggen, J. M., & Kirschner, P. A. (2003). Designing external representations to support solving wicked problems. *Arguing to learn: Confronting cognitions in computer-supported collaborative learning environments,* 177—204. Retrieved from http://edu.fss.uu.nl/medewerkers/ja/Confrontations/Ch7.pdf

Van Bruggen, J. M., Kirschner, P. A., & Jochems, W. (2002). External representation of argumentation in CSCL and the management of cognitive load. *Learning and Instruction, 12*(1), 121–138.

van Drie, J., van Boxtel, C., Jaspers, J., & Kanselaar, G. (2005). Effects of representational guidance on domain specific reasoning in CSCL. *Computers in Human Behavior, 21*(4), 575–602. doi:10.1016/j.chb.2004.10.024.

Vergnaud, G. (1998). A comprehensive theory of representation for mathematics education. *The Journal of Mathematical Behavior, 17*(2), 167–181. Retrieved from http://www.sciencedirect.com/science/article/pii/S0364021399800573.


UNCORRECTED PROOF

EDITOR'S PROOF


White, T., & Pea, R. (2011). Distributed by design: on the promises and pitfalls of collaborative learning with multiple representations. *Journal of the Learning Sciences, 20*(3), 489–547. Retrieved from http://www.life-slc.org/docs/White_Pea_MultipleRepresentations_2011.pdf.

Woolgar, S. (1988). Time and documents in researcher interaction: some ways of making out what is happening in experimental science. *Human Studies, 11*(2), 171–200.

Zemel, A. (2009). Reading's work in VMT. In G. Stahl (Ed.), *Studying virtual math teams*. New York: Springer Publishing.

Zemel, A., Koschmann, T., LeBaron, C., & Feltovich, P. J. (2008). "What are we missing?" Usability's indexical ground. *Computer Supported Cooperative Work, 17*, 63–85.

UNCORRECTED PROOF