

## Learning to learn together with CSCL tools

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### Abstract

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In this paper, we identify *Learning to Learn Together* (L2L2) as a new and important educational goal. Our view of L2L2 is a substantial extension of *Learning to Learn* (L2L): L2L2 consists of learning to collaborate to successfully face L2L challenges. It is inseparable from L2L, as it emerges when individuals face problems that are too difficult for them. The togetherness becomes a necessity then. We describe the first cycle of a design-based research study aimed at promoting L2L2. We rely on previous research to identify *collective reflection*, *mutual engagement* and *peer assessment* as possible directions for desirable L2L2 practices. We describe a CSCL tool: the Metafora system that we designed to provide affordances for L2L2. Through three cases in which Metafora was used in classrooms, we describe the practices and mini-culture that actually developed. In all contexts, groups of students engaged either in mathematical problem solving or in scientific inquiry and argumentation. These cases show that L2L2 is a tangible educational goal, and that it was partially attained. We show how the experiments we undertook refined our view of L2L2 and may help in improving further educational practice.

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## What is learning to learn together (L2L2)?

28 /Q5

The Learning Sciences constitute a domain in which ideologies often direct decisions about objects of research. This is the case of Learning to Learn (L2L), which is often referred to by governments and large organizations as the most important knowledge-age skill since it equips people to adapt flexibly in a time of rapid change (e.g., OECD 2001, 2004). L2L is a set of *capacities* and *meta-strategies* that help the individual learner face challenges for which he/she has to be specifically prepared (e.g., Claxton 2004; Fredriksson and Hoskins 2007; Higgins et al 2006). L2L fits a liberal or even capitalist ideology that prepares the individual learner to be an autonomous explorer in a changing world. Within this framework, Deanna Kuhn provides a conceptual definition of L2L. In *Education for Thinking*, Kuhn (2005) distinguishes between two types of skills that circumscribe Learning to Learn: Learning to Learn means developing *skills of inquiry*, and *skills of argument*. The boundary drawn by Kuhn and her distinction between the realm of inquiry and of argumentation is fundamental. Inquiry consists of procedures for apprehending the realm of experience and drawing proper conclusions. Argumentation is a central tool for the construction of knowledge. Many studies have demonstrated how to develop inquiry skills and argument skills separately.

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Two caveats stand in front of these important successes. First, the term “skill,” which is so important in Kuhn’s enterprise and in OECD’s rhetoric, stands in tension with the ambition to educate children to be part of a changing world: The term “skill” points at tradition and not at change. In traditional education, more experienced people guide the less experienced in particular skills. L2L aims at preparing the individual learner to be an autonomous explorer in a changing world. There is something presumptuous and at the same time traditional in this novel objective, the fact that the individual can be trained by instructors who know in advance the learning goals to be attained and the ways to attain them. But how can tradition help people change for a rapidly evolving world? Of course, it is possible to answer that for this very reason Learning to Learn partly consists of acquiring *meta-strategies* that are general enough to be applicable to situations that are totally new. However, the dubious results of research literature on transfer suggest that traditional teaching based on the learning of skills is not adequate.

The second caveat relates to the distinction drawn by Kuhn between inquiry and argumentation. While this distinction is important, the implementation of both of them in classrooms is not easy. In the EC-funded project ESCALATE, five teams from France, Greece, Switzerland, Israel and the UK implemented learning units interweaving inquiry and argumentation in science classrooms (Schwarz 2007; Schwarz et al. 2011). To ease the enactment of inquiry and argumentation practices, the teams capitalized on various software: microworlds for scientific inquiry and software for graphically representing argumentative moves. Although some local successes could be identified, comparisons between observations in all the learning sites made clear that the implementation was a failure. Reasons for the failure were multiple, but two were particularly salient. First, teachers had difficulties in orchestrating guidance in the context of 20 to 30 students in a classroom. Second, although they arranged students in small groups, different technological tools supported inquiry and argumentation activities and this separation made it difficult to reason/argue about experiences they had. The simultaneity of inquiry and argumentation activities could not be reached.

We saw, therefore, that although several programs have been successful in promoting L2L skills, the term “skill” constrains the learning to a legacy transmitted by competent adults and this constraint leads to missing the goal of preparing children to face new challenges in a time

of rapid change. Secondly, programs fail at promoting the two components of L2L – learning to inquire and learning to argue – in an integrated way. Technologies seem indispensable in this endeavor, but they have not realized this integration so far.

In this paper, we first present modest steps to address the two problems of L2L in the educational system. The first step is not new: The essential experience of the modern human is to evolve among other people who might be more experienced, but who do not know exactly how to face new challenges. Often, guidance is not available. Another ideology, more social, fits the difficulties people face in coping with change in modern times. Collaboration is a powerful instrument of this ideology. However, although people might join forces to help each other, the noble values that stand behind unity (solidarity, fraternity, etc.) do not help unless people know how to collaborate in order to face new and challenging situations. The need for this collaboration to face new challenges is ubiquitous in the 21<sup>st</sup> century: in the workplace as well as in the diverse organizations to which democratic countries enable citizens to belong. Productive collaboration on new challenges is a difficult matter, even for smart students (Barron 2003). As such, it becomes a new goal in education. The term Learning to Learn Together (L2L2) was first used by Rupert Wegerif based on work done with Marten de Laat (Wegerif and de Laat 2010). They conceived of a combination of the space and time of networks ('the space of flows' as defined by Castells, 2004) and of the space and time of dialogues (the 'dialogic space' as defined by Wegerif 2007) towards an overall approach for teaching higher order thinking skills in the network society. The Bakhtinian dialogic perspective was applied to networked learning of students to claim that an appropriate pedagogical design can support students learning higher order skills such as creativity and L2L (Wegerif and de Laat 2010). This very general claim served as a working hypothesis in the R&D EC funded Metafora project (Learning to Learn Together: A visual language for social orchestration of educational activities). Metafora focused on the design of a platform for supporting L2L2 in the context of solving problems in Mathematics and Physics. Our starting point in the project was to clarify L2L2, which was an unarticulated concept.

We saw in L2L2 an extension of L2L in the sense that it aims at promoting learning to inquire and learning to argue, as well as collaboration. We experienced that technologies are helpful for integrating inquiry and argumentation. The addition of collaboration as the third tenet of L2L2 naturally led us to posit that CSCL tools should facilitate L2L2 in group learning: Dedicated CSCL tools provide shared space for communication and co-construction of knowledge (Stahl 2006). They provide constraints and affordances for collaborative behaviors.

The concoction of learning how to inquire, to argue and to collaborate in the same pot sheds a new light on the essence of learning to collaborate. Traditionally, learning how to collaborate is understood to necessitate tasks that naturally lend themselves to collaboration, for example tasks whose accomplishment demands the assignment of different roles and different expertise (Rummel and Spada 2005). Such tasks are frequent in the workplace, for example in the management of projects, or in routine work in large organizations that demand high coordination of group work (e.g., between doctors and nurses from two teams during shift changes in hospitals (Engeström 2001)). Those who learn to be professionals must learn to be part of a team, to join forces with people who are different and have different expertise. Thus these participants have to learn to seek information, to ask for help, and to coordinate actions. The context here is a difference of roles and sometimes of status. It is uncommon in schools to see this type of focus, since schools tend to favour either equity amongst students or competition (to receive the best grades).

The context of learning to collaborate on challenging tasks is that of equal status, without assigning roles in advance to the members of the group. Collaboration emerges in inquiry or critical discussions, when the issues at stake are (too) difficult for individuals. Collaboration is then an *ad hoc* necessity. This kind of collaboration is inseparable from its object – inquiry or argumentation on a specific issue. Learning to collaborate among equals in order to handle a difficult learning task is a complex endeavour.

In this paper, the learning task concerns inquiry-based activities in science and in mathematics (it involves problem-solving in mathematics<sup>1</sup>). Collaboration serves these learning goals. It is not solely reducible to good procedures, but rather is always tinted by the experience of facing a challenge that is too difficult for the individual. Learning to Learn Together is then a considerable extension of Learning how to Learn, as the togetherness transforms L2L from an individual to a communal experience.

### Setting working hypotheses about how to promote L2L2 in classroom contexts

Although the three underpinnings of L2L2 – learning how to inquire, argue and collaborate were clear to us, our theory was unarticulated and our practical goals were fuzzy. We posited that since collaboration was central to L2L2, CSCL tools might be particularly helpful. Previous work in CSCL suggests that the design of the tools should encourage reflection and criticism (Fischer et al. 1993; Stahl et al. 2006). This vague situation is a classical starting point for design research cycles, in which it is hoped one will observe the emergence of desirable practices (Collins et al. 2004). Some of these practices are already known. The first group of practices consists of the heart of L2L2 practices – inquiry and argumentation practices. Scientific inquiry and mathematical problem-solving practices include raising hypotheses, collecting data or checking hypotheses (in scientific inquiry) and planning, solving a simpler problem or observing patterns in mathematical problem solving. Argumentation practices include for example elaborating arguments based on evidence, challenging or refuting. As previously mentioned, these two kinds of practices are learnable but their integration in classrooms is difficult. We aimed at developing a technology-based environment to afford the smooth integration of inquiry and argumentation. Since inquiry and argumentation practices set different goals among participants, we envisaged the interweaving of inquiry outcomes into argumentation practices.

L2L2 demands more. It demands to learn to collaborate while participating in inquiry or argumentation activities. It demands the establishment of a new culture by making existing norms explicit in order for groups to recognize them, reflect on them and be able to change them. As explained by Cobb and his colleagues (Cobb and Bauersfeld 1995; Cobb et al 2001), classroom norms are appropriated by enacting recurrent practices and making them public through discursive practices accompanying the enactment of the former practices. Two forces may facilitate or inhibit the emergence of this mini-culture. First, tools – especially communications technology – provide affordances for desirable practices. Secondly, the role of the teacher in L2L2 is complex: When engaging in inquiry activities or mathematical problem solving, students are not accustomed to collaboration. They are used to individual work: to

<sup>1</sup> Problem solving in mathematics and inquiry-based activities in science are structurally germane, although different traditions have been developed in science and mathematics. The most salient difference is that in science inquiry targets conclusions based on *experience* and/or on *theory* and that in mathematics, problem-solving targets *proofs* mostly relying on *deductive* steps. Another difference is that, in principle, in inquiry-based activities, the explorer sets his/her question, while in problem solving, the problem is given to the solver.

depending on the most competent student in the group. The intervention of the teacher to behave otherwise may be perceived as an intrusion in a well-oiled method of student interaction, prompting opposition from the students. However, it can be a dialectical process in which all participants (including teachers) behave as responsible actors, who negotiate within a specific context the norms and skills to be appropriated.

At first sight, Scardamalia and Bereiter already did the job. They showed that with Knowledge Forum, the creation by students of representations of meta-classifications of contributions leads them to an awareness of *collective agency* (Scardamalia and Bereiter 1999). The objectified community knowledge space is necessary for students' ideas to be objectified, shared, examined, improved, synthesized, and used as "thinking devices" (Wertsch 1998) to enable further advances. The general assumption that in order to take over high levels of social and cognitive responsibility, students' ideas must have an "out-in-the-world" existence, and that inventions, models, or plans, should be accessible as knowledge objects to the community (Bereiter 2002; Scardamalia and Bereiter 2006) is broadly accepted. It was exemplified by Lee et al. (2006), who examined the role of knowledge-building portfolios in characterizing and scaffolding collaborative inquiry guided by several knowledge-building principles. Students working on portfolios guided by knowledge-building principles showed deep inquiry and conceptual understanding. Also, Zhang et al. (2009) showed that Grade 4 students gradually *took collective responsibility* in a science classroom, meaning that each participant learned to take on interchangeable roles (leader, collaborator, helper, help seeker, etc.) and engaged in a way that advances the group (*epistemic advancement*). This pervasive phenomenon is encouraging. It shows the general feasibility of programs based on the use of dedicated technologies to help learning to collaborate on learning tasks. Scrutiny over what is meant by collective responsibility shows that it includes reviewing and understanding the state of knowledge in the broader world, generating and continually working with promising ideas, providing and receiving constructive criticism, sharing and synthesizing multiple perspectives, etc. (see Zhang et al for a complete list, p. 8). The importance of the idea of collective responsibility planted by Bereiter and Scardamalia is enormous. There is a need, however, to refine research in order to identify new relations between collaborative and knowledge components of learning. Our focus here on L2L2 – learning to collaborate while inquiring and arguing – is a step in this direction. In the next section, we will present a technological environment we developed to promote L2L2. The design of this environment relied on the kinds of practices we envisioned to be afforded by the environment. We already mentioned inquiry and argumentation practices as well as practices integrating both. Recent research on classroom learning helped us to identify three directions for collaborative practices that might promote L2L2.

The first direction – *collective reflection* has been recognized as important for constituting a community of learners in general (Yackel and Cobb 1996). As already mentioned, when students reflect on the ways they solve problems together, there is potential to highlight implicit norms that are not suitable and should be changed. While Yackel and Cobb were interested in teacher-led class discussions of past activities in elementary schools, there are other manifestations of collective reflection in secondary schools and higher education. In the context of inquiry/problem-solving of L2L2, we envisaged that an on-going collective reflection while planning and monitoring work together, facilitated by technological tools in which inquiry/problem-solving actions could be visualized and shared.

The second direction for collaborative practices relates to manifestations of *mutual engagement: help seeking, help giving and leadership sharing*. While help seeking and help giving are

easily identifiable as practices, leadership sharing is not translatable to definite practices. However, we encouraged teachers to prompt *shared leadership* when they thought group work was dominated by one student or when some students seemed idle.

A third direction for collaborative practice we envisaged concerns *peer group assessment*. Research has shown that both those assessing and those assessed can gain from peer feedback (Topping 2009), by spending more time on task and gaining a greater sense of accountability. It then can improve questioning and assessment of understanding through increased self-disclosure. Peer assessment is also found to increase students' interpersonal relationships in the classroom (Sluijsmans, Brand-Gruwel, & van Merriënboer, 2002). However peer group assessment rarely happens and is difficult to learn. As Dochy et al (1999) claimed, training in skills of group-assessment must be provided for a long time. Topping (2003) added important recommendations for learning group-assessment: among them, the fact that this learning should be communal, and should involve the assessed students. Kollar and Fischer (2010) added that peer assessment is often not a collaborative activity as it focuses on the assessors and is not addressed to the assessed. By using the term *peer group assessment* we clarify that to be included in L2L2, the assessment must be interactive.

To sum up, we explained that the promotion of L2L2 necessitates CSCL tools that support inquiry and argumentation as well as their smooth integration. Since inquiry and argumentation practices set different goals among participants, we envisaged the interweaving of inquiry outcomes into argumentation practices. In addition, we gleaned from the research literature findings on collaborative learning that point at types of practices to be learned indirectly – through affordances of the envisaged tool, or directly through explicit prompts of the teacher. We turn now to the description of the environment.

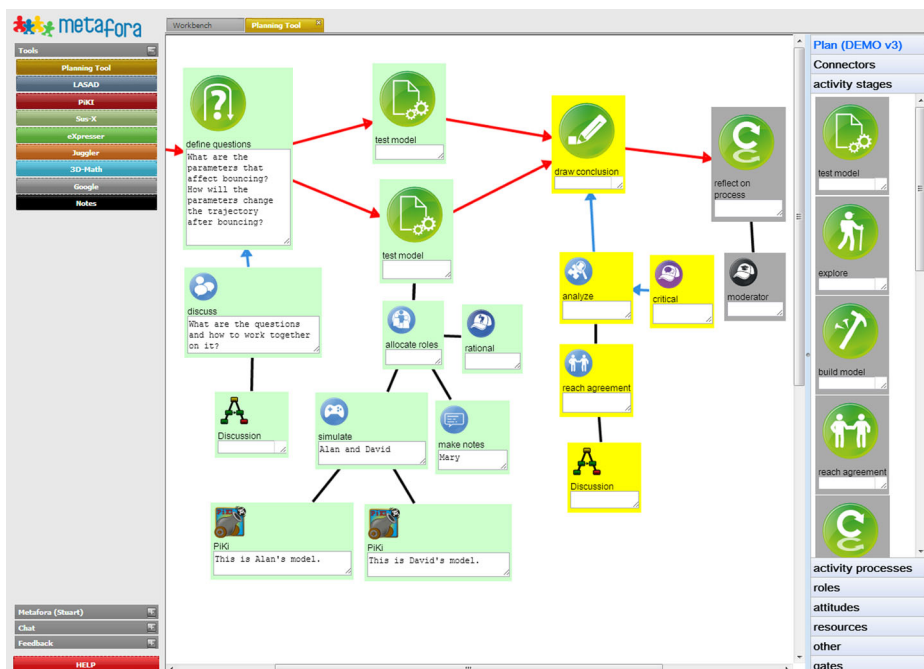
## The Metafora environment for promoting L2L2

The EU-funded Metafora project (ICT-257872) enabled the development of a system and of an educational environment aimed at promoting L2L2 (de Groot et al. 2013). The Metafora system comprises (1) a visual tool for planning and reflecting on group work, (2) microworlds for experiencing phenomena and exploring problem spaces, (3) a space for argumentation, and (4) a module for observing group work and possibly intervening by sending messages.

### The planning/reflection tool

The planning/reflection tool offers a visual language that enables students to create and map representations of their work for planning their activities and reflecting on them (see Fig. 1). Cards and connectors are available for this purpose. The cards contain visual symbols and titles, as well as space to insert free text (see Fig. 2). Some symbols and the titles represent different stages of scientific inquiry learning (e.g., the “explore” card in Fig. 2, or cards entitled “experimentation”, “building models”, and “hypothesizing”), and of mathematical problem solving (e.g., strategies such as trial-and-error, solving an analogous problem, or checking extreme cases). Other cards refer to group or individual processes (e.g., “discussing” in Fig. 2). A third category of cards represents roles played (e.g., “evaluator” and “critical” in Fig. 2). The fourth and final set of cards allows access to different resources within the Metafora toolbox (e.g., the card entitled “discussion” in Fig. 2 which allows access to the tool for structured discussion, or cards entitled “Piki” in Fig. 2 that serve as an entry point for a specific



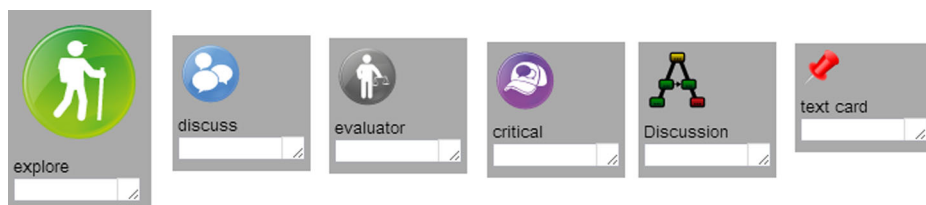


**Fig. 1** The planning tool. The *yellow* cards represent plans to be achieved; the *green* cards represent plans that have been already realized

microworld). The connectors (lines and arrows) represent relational heuristics (“is next”, “needed for” and “related to”) to explicate how the various cards are related in the given plan. The different features of the planning/reflection tool were designed to afford collective reflection on inquiry/problem-solving. Also, like for Knowledge Forum, posting stages of inquiry/problem-solving was planned to boost mutual engagement through the display of epistemic advancement.

## Microworlds integrated in the Metafora system

Five microworlds are fully integrated in the Metafora platform (Dragon et al. 2012; Kynigos and Latsi 2007). They serve as an arena for observation of scientific inquiry and experiencing mathematical problem solving. (1) *eXpresser*: a microworld designed to support students in generalizing algebraic rules by constructing animated models comprising patterns of repeated



**Fig. 2** Some representative cards in the planning tool

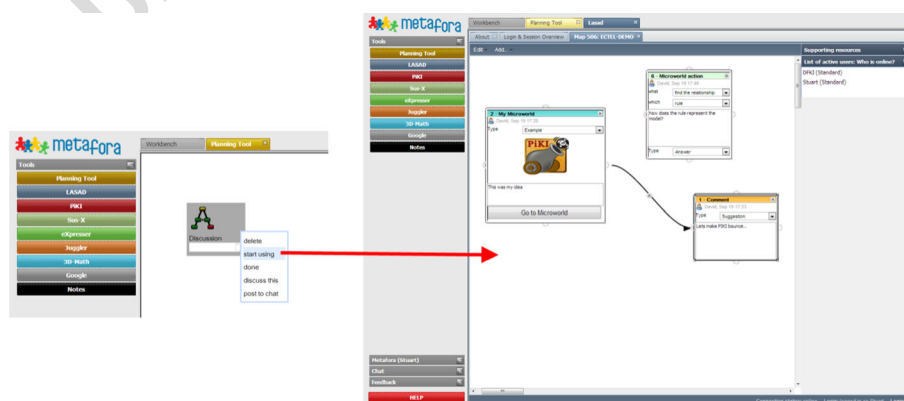
building blocks of tiles. (2) *The “3d Math” Authoring Tool*: a 3d programmable environment inside which users may graphically represent and manipulate 3d objects that they either find ready-made in an embedded library or construct themselves when using Logo procedures and commands. (3) The *“Physt 3D” Authoring Tool*: a 3d programmable environment that allows teachers to create 3d game-like microworlds for simulating phenomena defined by Newtonian Laws. (4) *Sus-City*: a game template for non-technical users (teachers and students) to construct and play their own “Sustainable City” games. (5) *PiKi*: a microworld that addresses kinematics through a serious game with a pirate-based theme. Other microworlds like Geogebra can be plugged in to the Metafora system in a less integrated way, but still allow productive collaborative mathematical problem solving.

## Discussion tools and referable objects

Metafora provides tools that allow students to engage in discussion and argumentation. The chat tool offers a quick and ever-present space for students to gain each other’s attention and share informal thoughts *in situ*. LASAD (Loll et al. 2012) enables the co-elaboration of argumentation maps (see Fig. 3). Both the chat functionality and the LASAD system are customized to bridge between argumentation and inquiry by displaying and offering links to *referable objects* that reside within other tools. These referable objects are generally snapshots extracted from the microworlds posted as components of the discussion. The referable objects enable discussants to defend or to challenge each other’s arguments, based on evidence collected/observed from experimentation with microworlds.

## Monitoring and intervention to promote L2L2 in Metafora

Much of students’ actions are observable within the software system. Monitoring of pure logged actions from users would quickly become overwhelming and unhelpful due to the amount of raw data. Therefore, an additional component of the Metafora system collects and analyses this low-level information, and produces higher-level information that is more helpful to human observers (Dragon et al. 2012). Students and teachers have access to this higher-level information, called *indicators*. The Metafora team elaborated a set of L2L2 behaviours, which



**Fig. 3** A LASAD map accessed through a discussion resource card in the planning tool. The map contains a referable object that represents a saved game in the PIKi game

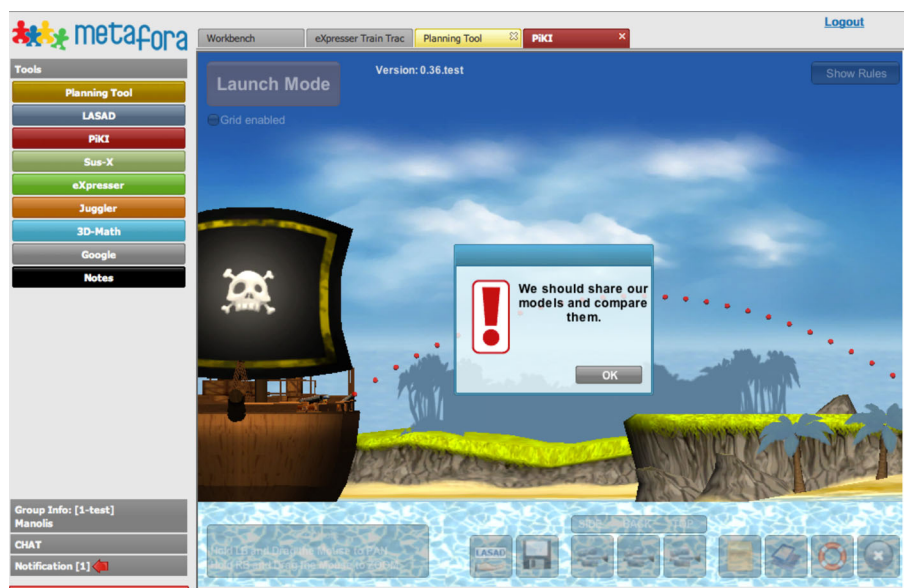


could be mapped to these indicators (or lack of) so that the system and observers could identify when certain behaviours were occurring. Simple examples of these behaviours that are identified by the automated analysis system include when all students in a group are contributing (a positive behaviour), or when a student is struggling, but not discussing their issues with teammates (a negative behaviour). Moving beyond identifying these behaviours, the Metafora team developed a tool to help sending messages that seek to remedy potential problematic behaviour. The Message Tool allows teachers and students to send messages to other students working with Metafora, and also allows the automated system to send messages in a similar fashion. This tool is populated with pre-defined messages (Fig. 4). Students and teachers can choose who receives these messages, and use the text as-is, or cater to the specific situation by altering the text before sending the message (Fig. 5).

Teachers and students can use Monitoring and Messaging tools in tandem to recognize problems and intervene to remedy the problems. However, the Metafora system goes one step further by using these tools together to offer direct intervention from the system to students potentially in need. For a simple example of an automated message, the system could detect that a long time and a number of other actions had occurred since there was any change in the plan, and therefore decide to send a message to all the group members stating, "How could we improve our plan? Let's look at the group planning map together". This message is taken directly from the pre-defined messages in the Message Tool (Fig. 5), as are all messages that are sent by the automated system. Each message in the Messaging Tool is linked with specific L2L2 behaviours that can be recognized by the system. Therefore, the automated system and students or teachers using the Feedback Tool are both encouraging L2L2 in the same way. The automated system contributes as much as possible with the currently recognized behaviours, and leaves the rest of the intervention task to the teachers and students.

The screenshot displays the 'Feedback' tool interface. At the top, there is a 'Feedback' tab. Below it, a prompt says 'Select any of the message templates from any of the tabs below'. There are five tabs: 'Lead Together', 'Share work', 'Friendly Feedback', 'Reflection', and 'Sent'. Under the 'Lead Together' tab, several message templates are listed: 'Let's propose a new idea to help us explore a different direction.', 'We need to see how the new ideas are relevant and helpful to our current work', 'How could we improve our plan? Let's look at the group planning map together', 'Let's assign tasks to help us split the work equally.' (highlighted in yellow), and 'Has everyone contributed in planning the work?'. Below the templates, there is a text input area labeled 'Type your message below' containing the selected message: 'Let's assign tasks to help us split the work equally.'. To the right of the input area, there is a list of recipients with checkboxes: 'Stuart' (checked), 'Bob' (checked), and 'Maria' (checked). Below the list are buttons for 'all', 'none', and 'edit'. At the bottom right is a 'Send' button.

**Fig. 4** The messaging tool. Students can adapt existing openers from different tabs (*top*) representing different L2L2 aspects. When one selects a message, it appears on the editing area for editing the message and selecting its recipients



**Fig. 5** Once a message is sent, it appears as pop-up anywhere that the students are working. In this case, a student is investigating their PIKI construction without much attention to the work of the rest of the group, and another student decides to send a message requesting the group to share and compare work

## Enacting L2L2 practices in educational institutions

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### Methodological considerations

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Following theoretical clarifications about L2L2 and the development of the Metafora system, the next step of our inquiry about L2L2 according to a design-based research approach was to implement learning units in different educational sites and to observe the emergence of classroom mathematical practices and learning processes. Based on *Grounded Theory* (Glaser & Strauss, 1967), we looked for regularities and patterns in the ways that the teacher and students act and interact as they complete instructional activities and discuss solutions. However, in contrast with Glaser and Strauss' approach, we had already developed the general categories of classroom inquiry-based and argumentative practices before we began the sample analysis that we present in this article. We also developed an environment that concretized affordances for L2L2. The general constituents of L2L2 – learning to collaborate, inquiry learning and learning to argue – were clear, but the specific activities involved were only envisaged. For example, we did not know whether the referable objects in Metafora, would help integrating inquiry and argumentation. Also, we did not know the specific practices involved in collective reflection or taking mutual engagement in the context of the promotion of L2L2. Therefore, we aimed at observing the specific practices that developed during the unit, and checking whether they met our requirements concerning L2L2. Accordingly, our research questions were: (1) What are the practices that actually developed in programs aimed at promoting L2L2? (2) How did the Metafora system mediate the enactment of these practices? and (3) Were these practices desirable with respect to our vision of L2L2?

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The analytic approach we took is interpretivist in the sense that we go beyond the observed social use of tools and symbols by inferring both the taken-as-shared intentions and meanings established by the classroom community and the interpretations that individual students make as they participate in communal practices. We describe here three cases in three educational sites. In each case, we describe the mini-culture established through the continuous use of the Metafora environment in consecutive activities and teaching actions intended to promote L2L2.

### **The first case: mathematics and collective reflection in problem solving**

The first case takes place in the course “Teaching and Learning in Mathematics Classrooms” conducted by the first author with his research team at the Hebrew University as a part of a pre-service program for mathematics teachers. The students were undergraduates in mathematics. The course was organized as a seminar of fourteen 90 min. long sessions in which pre-service teachers were introduced to major themes in math education. The theme “group mathematical problem-solving” covered 9 out of the 14 sessions. It included many topics: not only the classical focus on strategies, heuristics and their learning through metacognitive activity, but its contextualization into the framework of small group collaborative learning. In that respect, it had similarities with Stahl’s pedagogy for rediscovering Euclidian Geometry (Stahl 2013, 2016). Six students participated in the pre-service teachers’ program. All sessions took place in a computer lab. The sequence of the nine 90 min. long sessions on mathematical problem solving included among others: the invitation to enact the practices of collaborative mathematical problem solving, experiencing microworlds for turning problem solving into an inquiry process, ongoing planning in mathematical problem-solving, and the role of the teacher in collaborative problem solving. In the final session, which lasted 120 min, students were arranged in two groups of three; the first group posed a problem that the members of the group designed in advance; the second group attempted to solve the problem while the first group served as “teachers”. After 60 min, the groups swapped roles and the second group posed a problem challenge to the first group.

As a member of the Metafora research team, the teacher of the course was aware of the desirable norms. The teacher began the first session on group problem solving by explicitly expanding on a list of problem solving heuristics and strategies (taken from Pólya’s and Schoenfeld’s writings) and telling students that the first thing to do by teachers in the context of group problem solving is to identify the heuristics and strategies students adopt. The teacher also told his students that collaborating is very effective for solving difficult problems. He presented a list of practices of collaboration: practices of collective reflection, practices that expressed the taking of mutual engagement, and peer group assessment. In other words, the teacher clearly articulated the collaborative components of what we envisioned to be the aim of L2L2. He then arranged the students in two groups of three students. He told them that the first group would be invited to solve a problem and that the second group would be invited to observe them, then to report on them by assessing the quality of their solution and their collaboration. The teacher announced that the students would then swap roles on a second problem. During this session, one of the students, Ram, was dominant, pushed his peers to his [wrong] direction, and yet, was reluctant to collaborate with his peers.

In most of the following lessons, the teacher enacted the same stages: problematization, ongoing planning, accounting to the group about the solution path adopted, capitalizing on methods (heuristics) that previously succeeded in further challenges, experiencing possibilities

through the use of microworlds, and using analytical tools to prove a way to (be) convince(d). 378  
 To enact these stages, the teacher and students participated in many practices in which 379  
 Metafora tools were involved: 380

- a. Constructing a plan with the visual cards proposed in the Metafora environment as 381  
 constituting the ontology of components of problem-solving 382
- b. Sending messages with using the Message tool in Metafora (by the teacher to the students) 383
- c. Discussing solution paths through the Lasad tool 384
- d. Experiencing a phenomenon with Microworlds then building/rectifying the plan 385
- e. Reporting on a problem solving activity with the visual cards 386

In each session, one challenge was at stake. We took into consideration the high level of 387  
 these university students in mathematics, and we asked daring problems that required the 388  
 students to join forces and to think about methods to face them. Examples include: “How 389  
 many pieces can one get by cutting a big piece of cheese (planar sections) five times?” “How is 390  
 it possible to find the minimal distance between two boats sailing at a constant speed in 391  
 different directions?” or “How is it possible to find a place such that the sum of its distance to 392  
 three given cities would be minimal?” We discuss here the story of some moments involving 393  
 Ram – the student who dominated his peers in the first session of the course, in his relations 394  
 with his two peers while solving a problem during the last session of the course. 395

The scenario of this session replicates the scenario of the first session, with the difference 396  
 that at the first session, the teacher imposed the scenario, and at the last one, groups functioned 397  
 autonomously – the students designed their own activity. The teacher intentionally did not 398  
 attend the session; the students in each group gathered around one computer and communi- 399  
 cated orally. They used the Metafora Planning tool to report on and to plan the solution path; at 400  
 the same time, the students who played the role of teachers could send messages from the 401  
 message tool, and the students could also use the Geogebra microworld. Most of the time, one 402  
 representative from the group of “teachers” stood near the “students”, watched their work and 403  
 gave advice, while another one watched the groups’ work through the computer and sent 404  
 messages to the group through the message tool. We report on the challenge designed by 405  
 Group 1, in which students were designers and teachers, while the students in Group 2 played 406  
 the role of solvers. Group 1 included Irene, Livnat and Corine, three female students. Group 2 407  
 included two male students – Ram and Walid, and one female student, Rose. The challenge 408  
 was entitled: “Is 64 equal to 65?” In a first stage, Group 1 presented the two assemblages in 409  
 Fig. 6 to Group 2: 410

All shapes of the same colour seem congruent so that the rectangle and the square seem to 411  
 have the same area. However, when computing the two areas it appears that the area of the 412  
 rectangle is  $13 \times 5 = 65$  while the area of the square is  $8 \times 8 = 64$ . Group 1 prepared a series of 413  
 hints beforehand to help overcome the contradiction: 414

- Hint 1 Identify the sides whose magnitudes are certain, and the sides whose magnitudes can 415  
 only be inferred. 416
- Hint 2 Use scissors to assemble the parts in the square as a rectangle 417
- Hint 3 Use properties of similitude between triangles 418

They also prepared a proof to resolve the contradiction based on similitude of triangles. 419  
 Group 1 prepared then a second phase in which Group 2 was asked: “Is it possible to construct 420

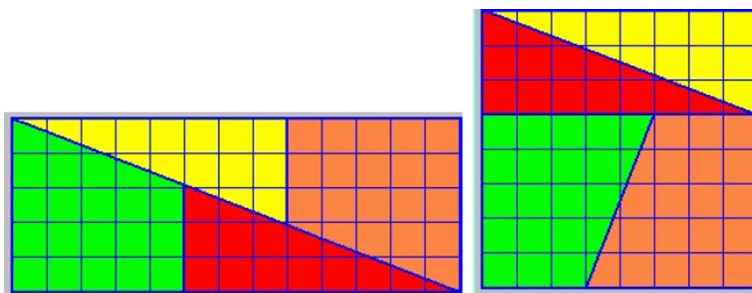


Fig. 6 Is  $64=65$ ?

another square of another size which leads to a similar contradiction?” Group 1 also prepared a series of hints: (1) to present a square and a rectangle whose sizes are  $55 \times 55$ , and  $34 \times 89$ ; (2) to present a square and a rectangle whose sizes are  $3 \times 3$  and  $5 \times 2$ ; (3) To construct an EXCEL table with the respective sizes of the sides of the square and of the rectangle and to find a pattern. These hints were designed to lead Group 2 to identify that sides arrange in a Fibonacci series. The hints encourage adopting strategies (solving similar problems, setting up a table, finding patterns) and suggest that Group 1 appropriated a culture of problem solving.

We describe here the first steps of Group 2’s solution. First the students of Group 2 read the problem. Then, Walid and Rose began working on the problem on their own by drawing the square and the rectangle with the different shapes on paper. In parallel, Ram began constructing a Metafora plan as a way to initiate the work plan of his group. He chose the card “Understanding the problem”, and then the card “Drawing a sketch”:

Ra: The problem is: We take a square of  $8 \times 8$  cm and cut it into some pieces. When putting the same pieces in a rectangle we get an area of 65 sq cm.

Ro: Yes, and how could it be?

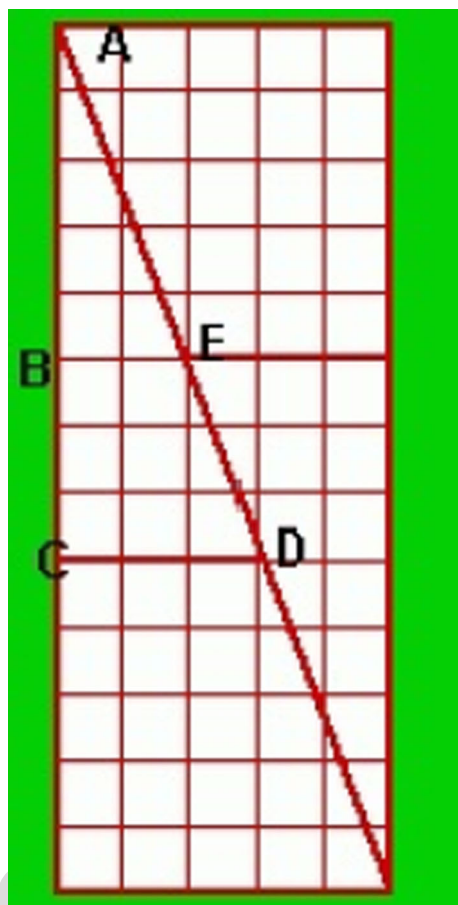
Ra: [hesitates, then inscribes in the card “how could it be?”]

Ro: [checks what is written on the screen] so you wrote it? and now sketch it!

Ra: [Opens a “sketch” card in the plan while Rose goes back to her notebook papers and sketches the rectangle].

We see that although the three members of the group work separately at the beginning, the fact that Ram decides to inscribe their on-going plan with the Metafora tool leads Rose to put her attention on his plan. After she adds *so you wrote it?* to the plan, she then mandates *And now sketch it!* This “order” leads Ram to bring a “sketch” card and to start to download GeoGebra – the suitable microworld to check the lengths of the segments of the problem. For Rose, this means starting a pen-and-paper sketch together with Walid. However, Ram’s attempts to download GeoGebra fail due to communication problems. He turns to geometrical considerations. He draws a figure (Fig. 7) and writes  $AC=8$ ,  $CD=3$ ,  $AB=5$ ,  $BE=2$ . Since the two triangles ABE and ACD are congruent,  $AB/BE=AC/CD$ , hence  $5/BE=8/3$ . Thus,  $BE=15/8=1.875$ . This effort shows him that there is “something wrong”. But he does not share with his peers this finding. Rather, he joins Rose and Walid to see how to arrange the different shapes in the square and the rectangle.

At this point, Ram turns again to the Metafora plan and informs Rose and Walid about joint work he reports in the “sketch” card: *We cut the square that we got and put them in the*



**Fig. 7** Illustration of Zvi's figures during his geometrical manipulations

rectangle. This is a ubiquitous reflective practice mediated by the Metafora plan tool. He then seeks another process card and chooses the “trial and error” card. But Group 1 observes this plan, understands that Group 2 is stuck and sends the message: *The lengths of which parts of the shapes do you know for sure?* through the Message tool. Ram who sits by the computer reads aloud the message, turns to the plan, adds the “Mathematical Model” card and inscribes in it: *We know the area of the triangles*. While writing, he hears Walid saying: *I don't think that there is an equation to calculate the areas*. Ram interrupts Walid and says:

They didn't ask us what we can calculate they only asked us which of the shapes we know for sure. I think that we know the trapezes, the trapezes we can calculate. [Points on the material figures]. Here you have  $\frac{3}{8}$  this is also  $\frac{8}{?}$  (on the other square, he counts) no/yes this is also  $\frac{3}{8}$  so it must be equal 24 cm. right? [Ram opens GeoGebra (successfully this time) and continues to lead the work of the group].

It seems that Ram who is the more knowledgeable in mathematics uses the plan for reporting about the group's work as part of the responsibility he takes with leading the group



towards the right solution. The plan is perceived as an arena representing the work of the group. Ram coordinates the construction of the plan. At the same time, he both leads and follows the group's work. The interplay between leading and following group work is best seen when Ram who describes the mathematical model card, receives with his peers the message of Group 1, and interrupts Walid's interpretation "they didn't ask us to calculate" [...]. At this point Ram abandons his suggestion to start with the triangles (which apparently could support a better solution path, showing that the two triangles are not congruent) to join Rose and Walid to guarantee that they are attuned for reaching a common solution. It appears then that the plan and the support of the message delivered by the message tool support *collective reflection* to bring all players to the desired joint solution.

As already mentioned, we concentrate on the mini-culture of this course. The excerpts we presented belong to the last activity of the course. We do not compare these excerpts with the ways the group collaborated to solve a difficult problem during the first session. However, it is noteworthy that Ram's behavior is very far from his behaviour in the first session, when he did not listen to his peers' ideas and led them astray down the wrong solution path. Rather in this later session, there is a clear manifestation of *mutual engagement* in the group, as Ram departed from a power relation where peers were paralyzed by Ram's capacities, and their work became more distributed as a result.

We add here another moment of the last session in which the role of the message tool was determinant in this acculturation. In parallel with Ram, Rose, and Walid's efforts to solve the challenge  $64=65$ , Group 1 (the triad playing the role of teachers) sent them messages through the message tool. During 8 min, Walid and Rose explored the trapezes and the triangles in pen-and-paper attempts; Ram pondered on a plan of action. He did try attempts with GeoGebra. Group 2 received then the message "consider others' ideas". Ram, who took the role of reporting on the group's efforts in the plan, reacts to the message:

Ra: Others! Do you have ideas?

Ro: [laughs]

Ra: Walid, do you have something to say?

W: Well I'm not sure if this is precise.

Ra: Walid do you have something to say?

W: I think that it turned out to be a whole rectangle [points to the 65 sq cm rectangle] because with the small parts here and there [points with his pencil to the square's composing shapes within Roses' sketch] we weren't precise enough [points to the sketch].

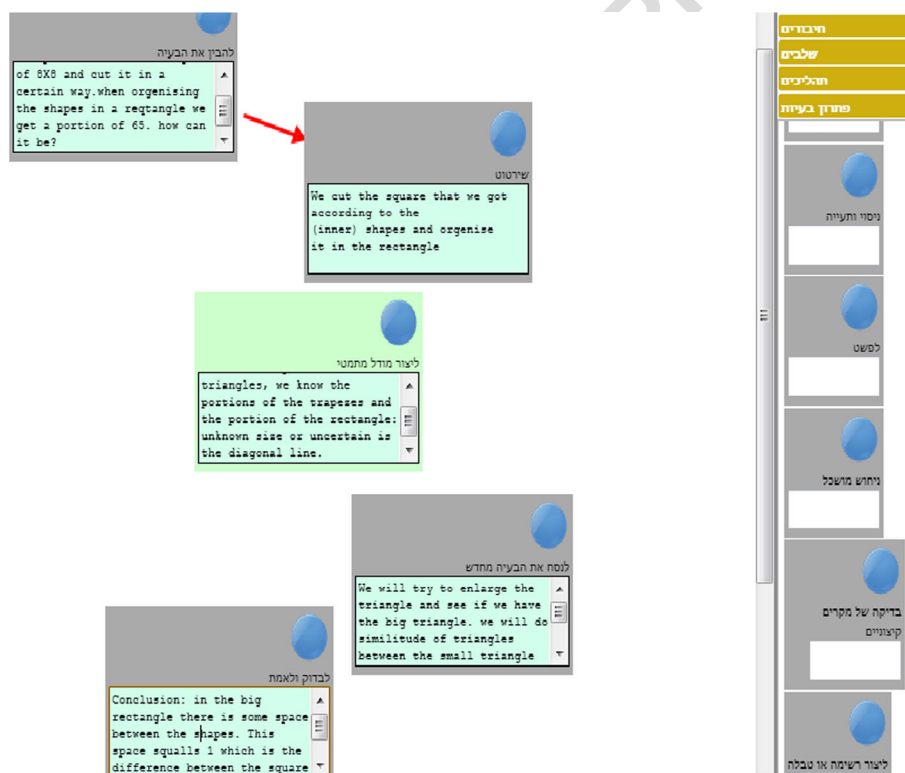
Ra: Here, you say? [Looks over the small pieces of papers] Here, it might be a problem with the way we cut it. [Turns to Walid] Here? Show me the place? [Looks over Walid's drawing]. No, here there is no problem. It's hard to know where there is a problem with the way we cut and where there is a real problem. So how can we do it? ahh....

Ro: Hmm... let's try to draw it in the GeoGebra. Then it will be more accurate.

Ra: We'll do the two shapes [the triangles] first.

It seems that the message arrived at the right moment when the group was pondering around with uncertainty regarding their next move. Despite his ironic reaction (Others! Do you have an idea?), Ram takes the message seriously, and insists to hear Walid's idea. But Ram is not satisfied by Walid's fuzzy explanation. He tries to understand what Walid is doing. His sincere attitude triggers Walid to verbalize his insight to the group by claiming that there must be some missing parts in the  $65\text{ cm}^2$  rectangle and that these missing parts might originate

from a lack of precision when using scissors for cutting shapes. Walid's claim triggers Ram to ask him to show where these missing parts might be (*here, you say? [...] Here? show me the place*). Ram then tries to continue Walid's idea. But he is puzzled (*where there is a real problem?*). This brings Rose (and Ram) to turn to GeoGebra for being more accurate. It appears then that the message provided a moment of consultation that pushed the group to another strategy – experimentation with GeoGebra (with which shapes can be measured more accurately). This strategy was adopted as a result of taking a mutual engagement: from Walid's idea which problematizes the challenge, to Ram's insistence to understand this idea, via Rose's suggestion to handle the problem as it is revealed by Walid's idea, and Ram's decision for action that complies with Rose's suggestion. We do not analyze here the further efforts of the group. However, Fig. 8 displays the on-going plan inscribed by Ram which represents a faithful report of the collective problem solving. We can see in Fig. 8 that Ram uses the "conclusion" card in which he writes: *in the big rectangle there is some space between the shapes. This space equals 1, which is the difference between the square and the rectangle*.



**Fig. 8** The report of Zvi in the plan see also legend below. Legend: 1. Understand the problem: The problem: we take a square of  $8 \times 8$  and cut it in a certain way. When organizing the shapes in a rectangle we get a portion of 65. How can it be? 2. Sketch: We cut the square that we got according to the (inner) shapes and organize it in the rectangle. 3. Create a mathematical model We know the portion of the triangles, we know the portions of the trapezoid and the portion of the rectangle: unknown size or uncertain is the diagonal line. 4. Reformulate the problem We will try to enlarge the triangle and see if we have the big triangle. We will do similitude of triangles between the small triangle ( $8 \times 3$ ) and the big one ( $8 \times 15$ ). it is clear that there isn't any similitude. 5. Check and validate Conclusion: in the big rectangle there is some space between the shapes. This space equals 1 which is the difference between the square and the rectangle

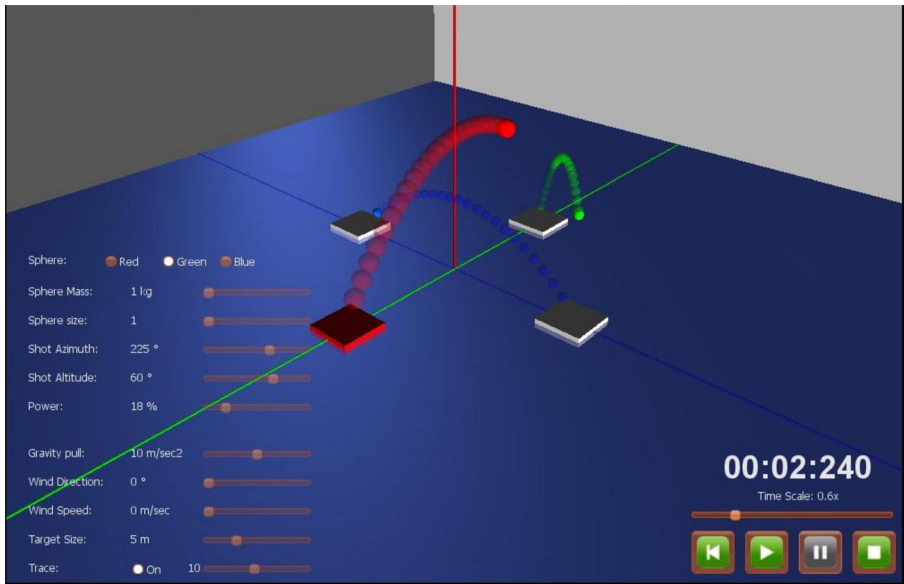
The Metafora platform supported then the emergence of a culture of problem solving (through the visual cards) as well as behaviours of mutual engagement through the message of teachers sent on the fly that identified an individual (Walid) idea that can be shared by the others. The Planning tool in Metafora crystallized collective reflection among students who could refer to their previous moves and plan their next moves. Although the Metafora environment fulfilled most of our expectations, we identified some undesirable behaviours: Ram elaborated geometrical considerations that showed that the calculation of length of one side (BE) through two methods yielded to different measures, but did not share this important insight with his peers. He preferred to follow the efforts of his peers with the material pieces cut with the scissors. This behaviour shows Ram's solidarity to his group but impairs the advancement of the group solution.

### **The second case: facilitating teachers' scaffolding of L2L2 in inquiry activities in physics**

The first case involved pre-service teachers interested in learning pedagogies. For them, the Metafora tools and the meticulous design of the activities were almost sufficient in their group problem solving: the teacher intervened rarely when peers interacted. The second case involves adolescents. We will see that the teachers are very active and that Metafora system is even more necessary to facilitate their efforts to support group inquiry and argumentation. The case takes place in a high school in a large city in Israel. The school provided additional hours for advanced students in science. Twenty three Grade 9 students participated in the study. The students were divided into two groups of 16 and 7 students each. The groups participated in a 1-year long course based on weekly 90 min. sessions in which the Metafora environment was used extensively. Typically, students worked in groups of 2-4 peers whose constitution was often but not always assigned. In most cases, students in the same group sat close to each other, with each student at an individual computer. Groups generally did not interact with each other, but individuals sometimes happened to ask for new ideas from other groups. During the first 2 months of the study, students were introduced to group inquiry and argumentation, to what we hypothesized to be L2L2 principles, and to the functioning of Metafora. From that time onward, students were presented with challenges. Generally, the students engaged with a particular challenge for 1-2 sessions. The students worked on activities in mechanics around the lever principle and laws of ballistics. The teachers met with the second author and our research team to design challenges in 3-4 consecutive sessions. Challenges were often directly linked to the microworlds integrated in Metafora. We focus here is on a group of three students in a challenge based on the Juggler Microworld (see Fig. 9).

Students were first presented with stroboscopic snapshots (Fig. 10). Students were asked to describe the motion captured in the stroboscopic snapshots. The students were guided to generate concepts relating to motion, in particular the concept of velocity.

Figure 10 shows a tennis player during a serve. The challenge that their teacher posed was "to identify the motion represented in the photograph, and to characterize it". The students organized in small groups and began discussing the photograph. The students reached agreement on the part of the trajectory that seemed to them the most simple – the movements of the tennis player. They went on investigating and discussing further aspects of the photograph. After several minutes, the teacher invited the groups to continue their discussion with the LASAD tool. The students broadened the challenge and deciphered as many aspects in the photograph as they could. They tried to figure out whether the tennis player was a man



**Fig. 9** The Juggler microworld. On the *left* side, one can see the sliders, which the users may use to change the parameters of the balls. Users may see the position of the ball in x, y, z coordinates by putting the cursor on the ball

or a woman, what the orientation of the racket is when it hits the ball, when the motion is faster and when it is slower. They did their best to be able to reconstitute the serve, taking into account details such as whether the hand passes between the player and the camera or on the other side of the player. Following the LASAD discussion, students were asked to plan an inquiry activity leading towards a deeper exploration of the photograph: they were asked to follow the ball in the snapshot and to characterize the velocity at each stage. For this endeavour the students were asked to divide the photograph in different parts and to assign roles within the group on how to execute their work exploring the motions of the ball and the racket. After finalizing their plan the students were asked to share their work to the other groups in a plenary discussion guided by the teacher. Once the sharing session concluded, the teachers decided to concentrate on exploring the motion in a better way as the use of the Edgerton stroboscopic picture proved to be too complicated for achieving this goal. Then students received a new challenge about finding the mathematical equation allowing them to plan where to situate an alarm system in case of a missile attack, an unfortunate situated task in the Middle East. For this purpose, the students used the 3D Juggler microworld to explore ballistic motions as Juggler made time, distance and velocity of bodies moving in the space visible to students.

We focus here on the first stages of the work of Ely, Sami and Yaron – working together on the Juggler challenge, first on their LASAD map. Figure 11 shows a clear disagreement between Yaron and Ely regarding the way the tennis ball reaches the player's racket: Ely thinks that the player first hit the ball on the ground; when the ball bounces to the same height and starts to fall down, he hit it. Yaron claims that the ball does not reach the ground and that the player throws it up, and then hits it. He also challenges Ely's claim that there is no sign that the ball reaches the ground. Interestingly, Ely at this stage does not try to answer Yaron's

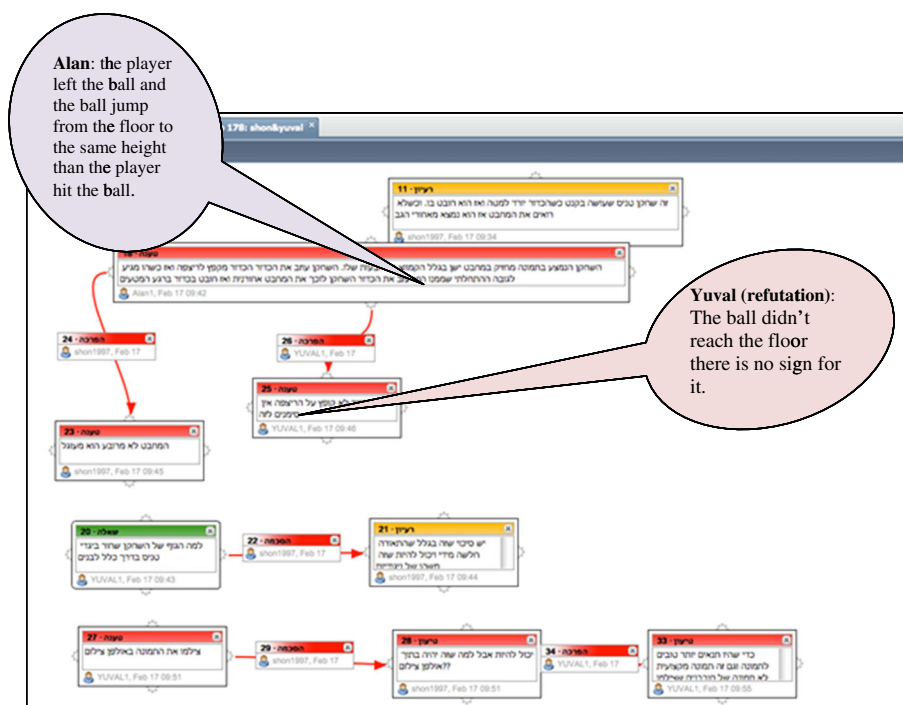


**Fig. 10** The stroboscopic photo presented to the students for exploration

challenge; rather, he contributes to the ongoing discussion on the lighting conditions that affected the photos of the tennis player and exact measurement of the racket. The Lasad map of the discussion shows that students presented to each other their interpretation of their part of the photo, leading by such to the interpretation of the picture as a whole. At some point, Yaron asks Sami whether he also sees the ball jumping on the floor, and Sami answers that he cannot see it. Figure 11 displays a partial view of the map (the full map includes 36 contributions).

Following the LASAD discussion, the students were asked to use their ideas to elaborate a plan for exploring the movement of the tennis ball with the planning tool. Students were given the half-baked plan appearing in Fig. 12. This plan suggests that each student should analyse a different part (as it appears in the card “allocate roles” and in the text of the cards “blank stage” and “build a model”). Each student sat by his/her own computer.

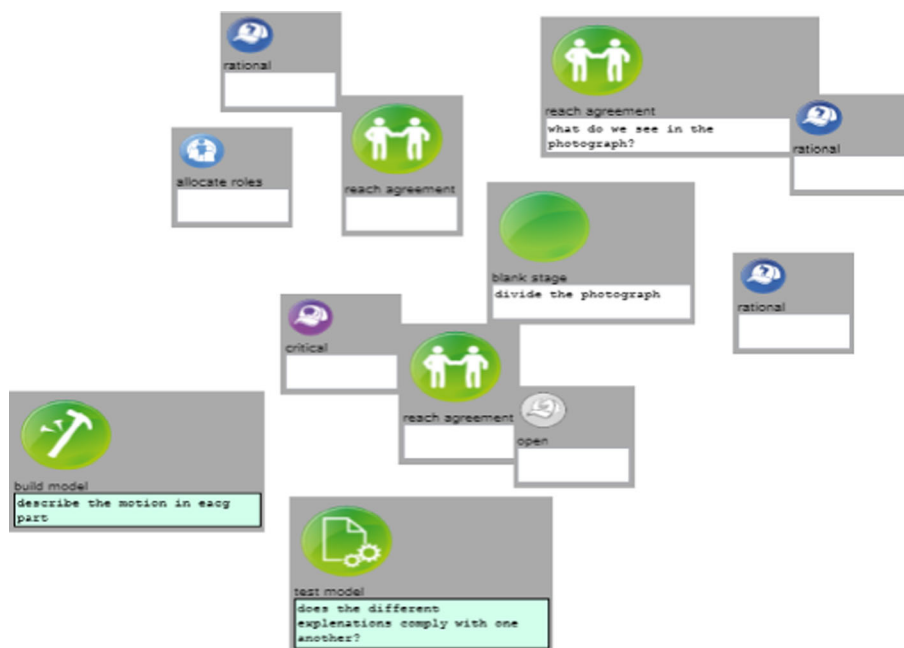
The triad used three cards only to fill the half-baked plan: (1) “Role allocation” (proposed by the teacher), (2) “pose questions” (not proposed) and (3) “find hypotheses” (not proposed). In the “role allocation” card they wrote: “Sami builds a hypothesis about the motion of the racket, Ely builds a hypothesis about the motion of the ball and Yaron builds a hypothesis about the player and poses interesting questions that should be answered”. Indeed, Ely raised reasonable hypotheses and Yaron elaborated interesting questions such as “how much time it took for the whole picture to be taken?” The students used some of the instructions embedded in the “half baked” plan, but decided to elaborate more towards inquiry-based learning – by adding a hypothesis card and by connecting it with a red arrow to the “build model” card.



**Fig. 11** A snapshot of the LASAD map of the group highlighting disagreement between Yaron and Ely on the interpretation of the stroboscopic photograph in relation to the tennis ball's move

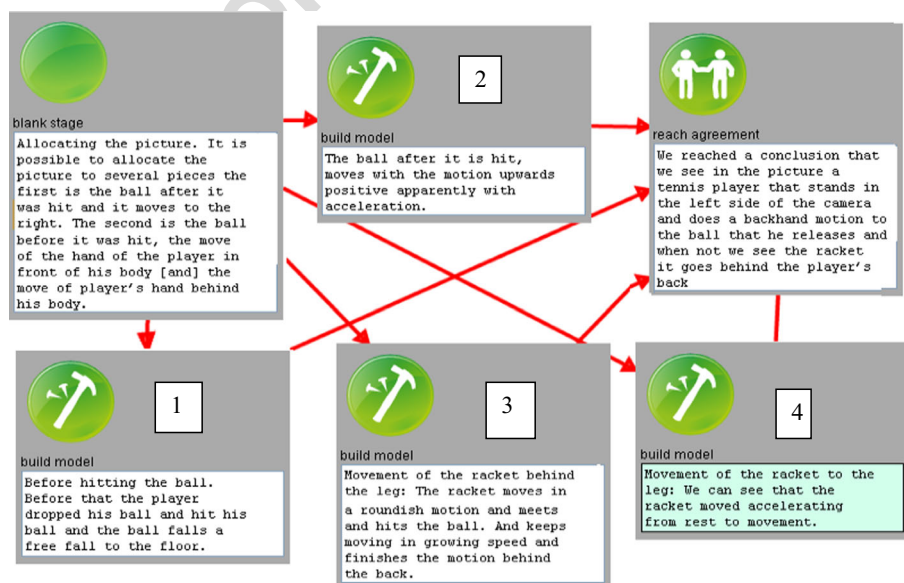
Although the students were sitting next to each other, they hardly spoke with each other, but rather filled their planning map to explain what they are about to do. When the teacher saw their plan in the next session, she asked them to turn it to more executive, and asked them to shorten the text in the cards. She pushed her students to carry a collective reflection over their first plan for producing a better plan. The triad changed the plan to what appears in Fig. 13. In the “Blank Stage” the triad explains how the photograph was divided in three regions: the trajectory of the ball before it is hit, and after it is hit, and the movement of the hand/racket. This new plan is based on the previous plan. When Eli describes his hypothesis about the movement of the ball he splits it into two phases: first, when the ball falls (he writes “the ball is in free fall, thus accelerates. It is possible to prove it based on the distances between the balls. Because of the growing distances it is reasonable to say that the speed increases”); secondly, when it is hit (he writes “Now for the movement after the hit... the ball accelerates. It is possible to prove it based on previous ways, and regarding the incline we see it in relation to the floor”). The second plan of the group (Fig. 13) shows that Ely beautifully describes the movement through “build a model” cards: the first describes the ball in free fall. The second is devoted to the movement of the ball after it is hit. The footprints of the role allocation inscribed in the first map are visible in the third and fourth “build model”: card 3 describes the movement of the racket behind the leg; card 4 describes the movement of the racket to the leg (“We can see that the rocket moved accelerating from the rest of the movement”). This





**Fig. 12** The “half-baked” plan given to groups

suggests that the group reflected over their previous plan towards the completion of  
their descriptions. It appears then that the provision of the half-baked plan by the  
teacher frames the further collective exploration of the photograph by the group.



**Fig. 13** The second plan of the group

Following the completion of the plan, the teacher asked the triad to share it with the rest of the class in the plenary. The students projected it on the wall. We report here on their presentation and on the role of the teacher in this presentation:

Yaron [Points to the map, to the “our stage” card] First, we allocated the map to areas [...] The ball, after being hit, moves to the right, the ball before it was hit, the motion is downwards. And the movement of the player’s hand behind his body, thus, backwards. There is a black part, which is probably his body, and then the player’s movement after the hit. (Points to the four “build model cards”) Now, we built models of every motion, here, here and here. [To Ely] now you read your model.

Ely [reads the text in the card] Humm... before hitting the ball, before the player dropped his ball and hit his ball and the ball falls a free fall to the floor.

Teacher OK, one moment. This...how would you define it...is it sure? Is it...

Yaron Yes, it is certain

Teacher Is it a conclusion? What is it?

Yaron Ok...there is here...” After the freefall” is certain [Points to Ely’s “build a model” card]

Teacher This is certain?

Yaron This is certain

Teacher OK. How do you know it for sure?

Yaron It’s the earth, gravitation...it exists

Ely There is some disagreement...

Yaron There is some disagreement about the ball...

Teacher No, who said that it is [shows with her hand the possibility that the ball moves up]

Yaron This is what we said, the part with the freefall is ok, but the part where he drops it, No [Points to the “build a model” card].

Ely Exactly!

Teacher So this is a hypothesis

Ely [...] There is earth. There is disagreement about the way the ball falls. There are several trajectories. Whether the player can drop it and then hit [moves his hands accordingly] or can throw it up or make it fall down and it jumps [back] and...

As in the first case, we see here that the Metafora plan which had been created to regulate and coordinate the group’s efforts, serves here as a tool for *collective reflection*. Yaron and Ely report on their collaborative work. They also report on their disagreement. This suggests that they collectively took responsibility in solving the problem, as they see this disagreement as something that should be further explored to allow agreement within the group. Moreover, the disagreement between Yaron and Ely cannot be settled without going deeper into the analysis of the movement of the ball. The teacher invited the triad to measure distances in the different positions of the ball in the different times at which the stroboscopic photograph had been taken (pictures were taken every 0.01 s.). But, in spite of the eagerness of the triad to learn together, they were stuck because they did not have the means to measure the exact movement of the racket and the ball only from the stroboscopic picture. At this point the teacher decided to invite them to experience motion and velocity with the Juggler microworld. To this end, the teacher created a new challenge related to daily life in the south of Israel: “You are a group of engineers that should design a device for alerting the population when a missile is launched from Gaza to Israel. You have to find an algorithm to calculate the place from where the

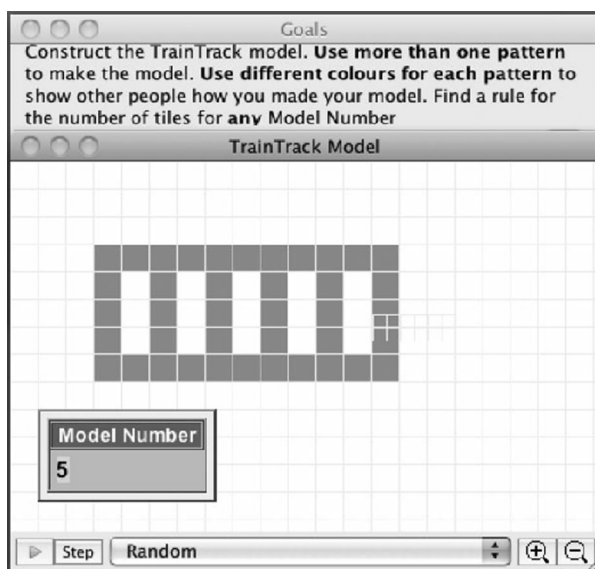
missile launched and where it is going to fall". For doing so, students were asked to use the Juggler microworld (see Fig. 9), and to plan their joint work with the planning tool. Juggler allows exploration of ballistic trajectories through a stroboscopic view similar to the Edgerton photograph: one can measure distances by putting the cursor over the positions of the ball and viewing its coordinates. Yaron, Ely and Sami worked together during two sessions: Yaron and Sami checked different positions of the ball (varying initial angle and velocity) with Juggler; Ely created an Excel table in which he collected data. The three students then used the table in an attempt to construct the equation. Since the elaboration of the formula representing the trajectory of the missile was a bit difficult, the students asked some guidance from the teacher. This request for help did not express dependence, but a timely need from their part in their intense mutual engagement in the task. We will return to the subtle role of the teacher in the concluding section. Let us mention only that her invitation to use LASAD afforded the surfacing of conflicting interpretations of the stroboscopic photograph on a common space. She also triggered group reflection through the planning tool, first by providing a "half-baked plan" for completion. We showed that the development of the plan of the group and its presentation in the plenary session were accompanied by questions the teacher asked to squander vagueness and to support students in expressing what they think on the ideas agreed by the group and by individuals. This role contributed to empower a mutual engagement evidenced when the group "shared disagreement."

### **Third case: emergence of peer group assessment as a result of the use of referable objects**

The third case is about the role of referable objects in group inquiry and argumentation. We already mentioned the shortcomings of not integrating inquiry and argumentation in learning tasks. We will see here how Metafora afforded this integration. The topic of the third case is early algebra. Although junior high-school students are generally fluent in manipulating algebraic expressions, they have difficulties understanding the concept of a variable as well as conceiving of and identifying algebraic structures (Rojano 1996). They need to develop *algebraic ways of thinking including* (a) perceiving structure and exploiting its power; (b) seeing the general case when presented with specific instances, including identifying variants and invariants; and (c) recognizing and articulating generalizations, including expressing them symbolically. The eXpresser microworld was designed explicitly to foster such algebraic ways of thinking (Mavrikis et al. 2011).

In eXpresser, students are presented with a model and asked to construct it using one or more patterns (see Fig. 14). The model is animated, with the Model Number – a variable that represents the number of holes in the pattern. The animation serves to emphasize the generality expected, instead of just counting tiles. The task is to find a rule fitting the number of tiles of any given model number. To construct the model, students are first asked to express how they visualize its structure as repeated building blocks. Students then make their rules explicit by defining an algebraic expression that calculates the number of tiles in each pattern. When the rule is correct, the pattern becomes coloured. Of course, there are multiple ways to fit a pattern. Figure 15 shows two possible ways of creating the pattern on Fig. 14.

The multiplicity of ways models can be constructed and the difficulties individual students have in their algebraic ways of thinking open the door to togetherness: Geraniou and colleagues (Geraniou et al. 2010) hypothesized that the equivalence of algebraic structures can be perceived in collaborative and argumentative activities in which students engage around

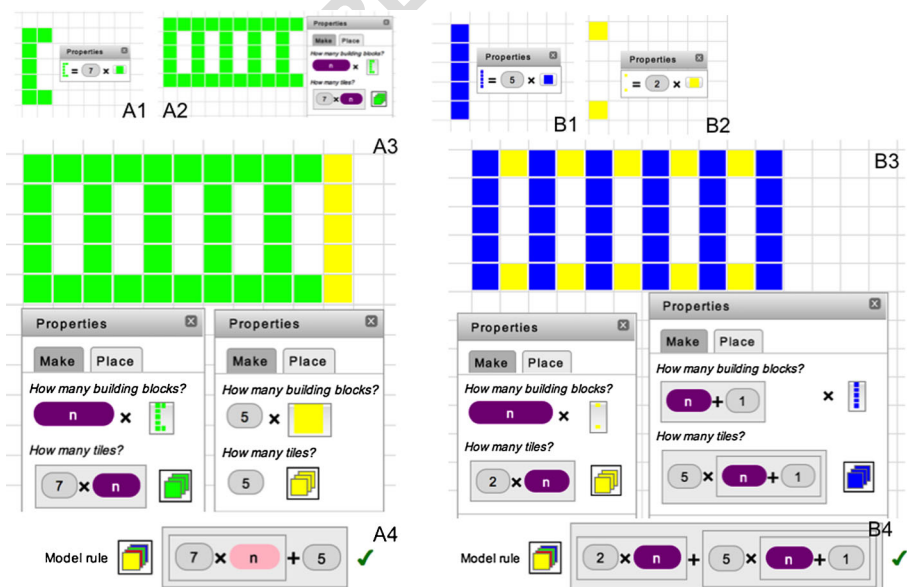


**Fig. 14** The 'Train Track' task in eXpresser. The model to be constructed is animated with the 'Model Number' (the number of 'holes' in this case) changing in random steps every few seconds

the correctness and equivalence of the algebraic rules that govern a certain model. This hypothesis naturally led to the integration of eXpresser into the Metafora system, and

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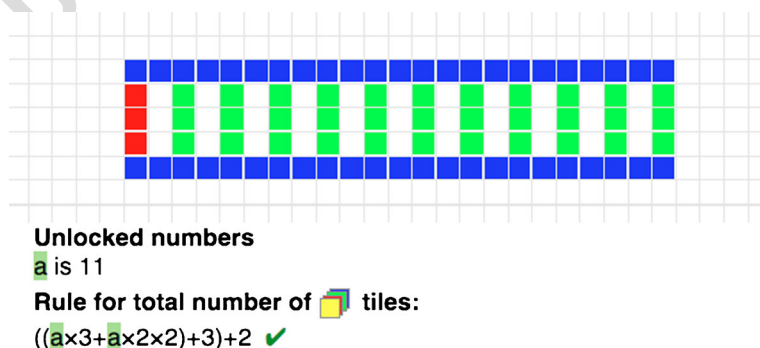


**Fig. 15** Two different models in eXpresser for the same figural pattern. The *left* one is made out of a building block of 7 green tiles (*A1*) repeated *n* times (*A2*) and a fixed 'column of 5 tiles' (*A3*). Therefore the corresponding rule for the total number of tiles is  $7n + 5$  (*A4*). The *right* model consists of two building blocks: one of 5 blue tiles (*B1*) and one of 2 yellow tiles (*B2*). For each yellow column there is one more blue so that the yellow columns are repeated *n* times whereas the blue  $n + 1$  (*B3*). The rule therefore for the total number of tiles is  $2n + 5(n + 1)$  (*B4*)

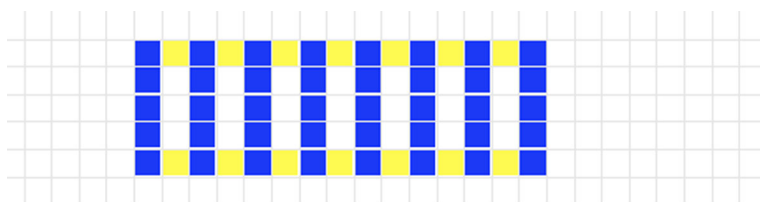
especially to the use snapshots and animations of eXpresser as referable objects in LASAD discussions.

The third case takes place in a school in the greater London area. Students from a mathematics class of 11 year-old students volunteered to undertake a Metafora challenge. The students are first introduced to L2L2 principles and to the functioning of Metafora. Students are then presented a challenge around eXpresser. They are invited to create the pattern displayed in Fig. 14 in eXpresser and to generate it in different ways. Students organize in groups and generate algebraic rules. The groups are then invited to use LASAD to compare findings with other groups, hopefully leading them to insights on algebraic equivalence. Two of the groups, the “Aristotle” and the “Fibonacci” compare their rules in LASAD.

Initially, the rules are not very clear but through the discussion in the chat, each group is asking for clarification. For example the first rule that the “Fibonacci” subgroup has shared is the eXpresser rule  $((a \times 3) + (a \times 2) \times 2) + 3 + 2$  but they haven’t made clear what ‘a’ represents (see Fig. 16). Mikis from the Aristotle group asks, *What about the blue blocks and the red blocks?* The Fibonacci group answers ‘a’ is the model number (i.e., the variable that represents the number of holes – see Fig. 16). In further clarifications, the Fibonacci group answers  $a \times 3$  [to find the blocks in the green bar] +  $(a \times 2) \times 2$  [to find the blocks in each horizontal blue bar] + 3 [the red bar] + 2 [the two blue blocks on top and below the red block]. One of the rules that the Aristotle group found is  $5 \times x + 2 \times (x - 1)$  (see Fig. 17). But some members of the Fibonacci group are not convinced that this rule is correct. Figure 18 shows a small part of a LASAD map in which Tom from the Aristotle group and Joel from the Fibonacci group argue with each other. In Box 7, Tom animates his rule – by such turning this animation to a referable object shared in LASAD; In Box 41, he comments that for  $x=1$ , a blue line only is generated. He disqualifies the rule in Box 49 in quite vague terms. Following this assessment, as shown in Fig. 19 which shows another part of the LASAD discussion between Tom and Joel, Tom modifies the rule from  $5 \times x + 2 \times (x - 1)$  to  $5 \times (y + 1) + 2 \times y$  (Box 51). The variable y represents now the number of yellow columns (and thus implicitly the number of gaps in the pattern) (not part of Fig. 19). This suggestion is certainly acceptable. It points at full compliance with the constraints of eXpresser, according to which the variable should be the number of gaps and should indicate a pattern. The rule  $5 \times x + 2 \times (x - 1)$  does not show any yellow column when  $x=$



**Fig. 16** The model of the Fibonacci subgroup. The model is made out of a building block of green columns repeated a times. The students also used a red building block for the edge and two blue tiles on top and below the red block). Then they used a horizontal building block of 2 tiles repeated also a times above and below the green columns



### Unlocked numbers

$X$  is 8

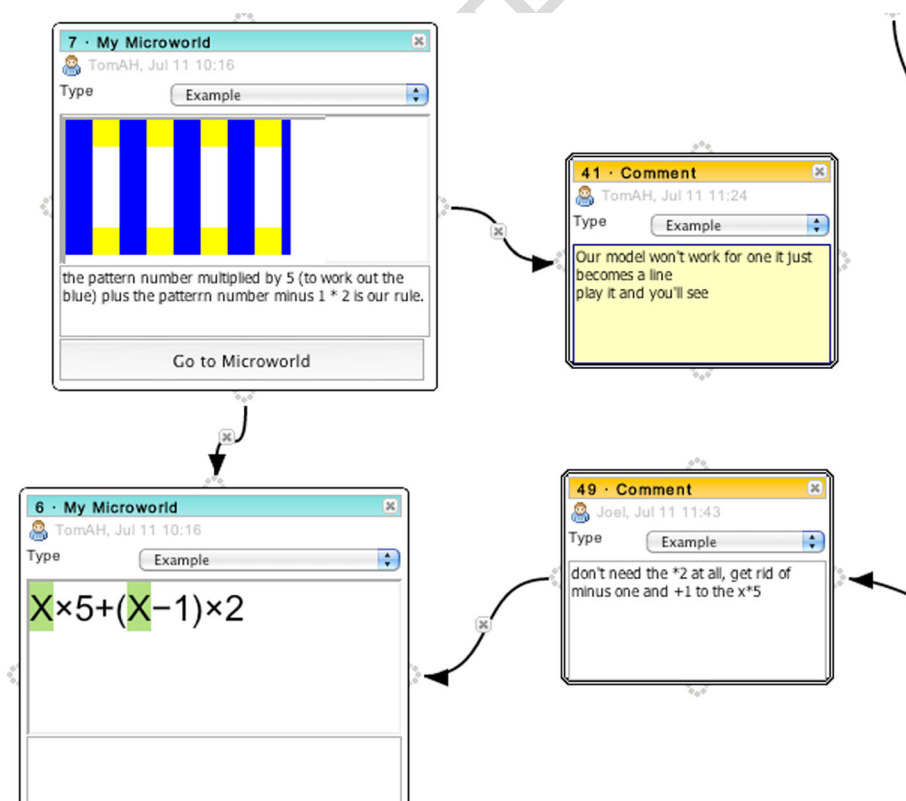
Rule for total number of tiles:

$$X \times 5 + (X - 1) \times 2 \quad \checkmark$$

**Fig. 17** The model of the Aristotle's subgroup:  $x$  is used to represent the number of 5-tile (blue) columns. This means that for  $x=1$ , there are only 5 tiles in the model – the first blue column. The yellow column is repeated  $(x-1)$  times and therefore when  $x=1$  it does not appear at all

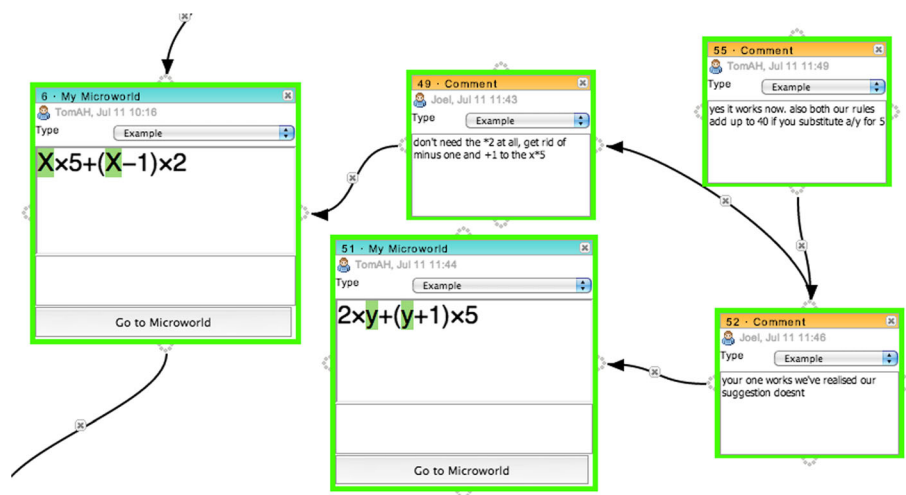
1, and by such, does not show any pattern to be repeated. However, it is fully acceptable from an algebraic point of view. The disqualification of the rule is then technically understandable but is mathematically wrong. The presence of a good teacher would have responded to this weakness.

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**Fig. 18** Tom from the 'Aristotle' group shares his model with Joel from the Fibonacci group who is critical to the rule Tom proposes and disqualifies it





**Fig. 19** Tom and Joel represent their groups in these LASAD contributions that help to refine the rule of the Aristotle group (box 51) and subsequently test it for equivalence against the Fibonacci rule (box 55)

The unguided interaction between the students led to an interesting conclusion, though. We see in Box 52 (Fig. 19) that the rule that “Aristotle” team shared (box 51) is not only approved by both teams but also leads to box 55 where it is being tested for equivalence with the rule of the Fibonacci team. This is because both teams have animated their rules and have evidenced that for  $y = 5$ ,  $2y + 5(y+1) = 40$  (Aristotle group) and for  $a = 5$ ,  $((a*3)+(a*2)*2)+3)+2 = 40$  (Fibonacci group). This justification strategy was a common way of mutual peer assessment. This kind of peer assessment is interactive and as such is more susceptible to lead to learning, as stated by Kollar and Fischer (2010). This interactivity of peer assessment was afforded by referable objects in Lasad discussions, which led groups to test and to correct each other’s rules through creating an approved model on the eXpresser microworld. Referable objects in LASA D maps then supported interactive *peer assessment*. We did not forecast this phenomenon. We recognized the necessity to integrate group inquiry and argumentation. This integration, which was realised through the insertion of referable objects of experience in argumentative maps, boosts interactive peer assessment for learning.

## Discussion

The three cases we presented occurred in a classroom with several parallel groups and a teacher whose rare interventions consisted of prompting collaborative behaviours. This fact is noteworthy (see another example in Schwarz and Asterhan 2011). It shows that something special happened, since unguided group work at learning tasks is generally unproductive (Webb, 2009). For all cases, groups engaged in inquiry, problem solving and argumentation. We could not present here clear evidence for progress, but the first case, which depicts the last session of a course, shows highly skilled students in designing/solving challenging problems. It is reasonable to think that the students progressed considerably in L2L. A comparison between the first and last sessions (that we did not present here to keep the length of the article reasonable) would have demonstrated such a progression. The Metafora environment

facilitated this progression as it provided a visual language for inquiry as well as for argumentation. Our ambition in this paper was bigger. We conceived of a more encompassing competence than L2L. In Learning to Learn Together, L2L is nested into collaboration. We envisioned several L2L2 practices to be implemented and we equipped Metafora with what we thought to be affordances of those practices. The three cases show the actual practices implemented. Some of these practices fulfilled our expectations, while others contained surprises. We list here these practices and stress the role of Metafora in their enactment.

### **On-going reporting on and planning of collective problem-solving/inquiry**

The efforts of problem solvers/inquirers working collaboratively or individually are punctuated with the inscription in some common space of the actions thus far completed. The Metafora planning tool served in the first two cases as the material space on which this inscription is done, through cards that represent strategies, stages, and procedures that represent actions and activities taken or foreseen to be taken by the group, both past and future. These cards mix problem solving/inquiry categories with categories of group work. As we saw, through such inscriptions, groups reflected on their work and planned future lines of action. Texts such as “How do we function?”, “What will we do or what did we do?”, and especially interjections such as “Why did you do it?” or simply “What” or “Why” suggest that the space created by the Planning tool affords *collective reflection*. In the first case, the planning also served as a material space to plan future actions. In the second case, the teacher sent a half-baked plan to the group of students – a way to boost collective reflection and mutual engagement. In both cases, the construction of the plan or the inscription of the report promoted further work.

### **Critical discussion of plans and past work**

This practice was omnipresent in the contexts of the three cases. It was embedded through LASAD, which provides tools for visualizing argumentation. Expressions such as “what did I see?”, “what is my interpretation”, “do I agree with my pal?” are typically typed in LASAD maps. In the second case, LASAD was used before or after the use of the Planning tool. Accordingly, students alternated their engagement as individuals and members of a group: individual arguments and opinions were accounted for and possibly challenged by the group, further plans and explorations were planned in order to reach an agreed solution to the challenge at hand. In the second case, a LASAD discussion allowed all students to discuss their opinions and arguments about the photograph. Following the teacher’s instruction to partition the picture and to explore each part separately, the students received a half-baked plan that supported their mutual engagement for converging on an agreed explanation about what was seen in the Edgerton photograph. The agreed explanation was criticized again during the plenary session, represented not as two different opinions within the group, but as a difference of opinions shared by the group. This shared difference of opinions was the vehicle for further co-exploration.

### **Experiencing mathematical/scientific phenomena and recording their deployment**

Experiencing mathematical/scientific phenomena with microworlds is not new. The second case (with the Juggler) and the third one (with eXpresser) exemplify that Metafora effectively affords experiencing phenomena.

## **Directly linking the experiencing of mathematical/scientific phenomena and critical discussions** 871 872

In the introductory part, we mentioned the disconnection between inquiry and argumentation. 873  
We have seen in the third case that Metafora offers the possibility to record critical moments of 874  
scientific experience as dialectic tools serving as evidence in further activity. This is what we 875  
saw in the third case when one student challenged his peer who counter-argued by sharing his 876  
experience with eXpresser – the animation of an algebraic structure. The import of referable 877  
objects in LASAD maps, enables the direct branching of experiences undertaken by students 878  
into their argumentation. As shown in the third case, this direct branching led students to assess 879  
what was claimed by their peers because they could directly test the evidence brought forward 880  
to support their claims (about an algebraic equivalence) in an argumentative process. 881

## **On-line adaptive intervention of the teacher in group work** 882

The Message Tool, with its specific openers, was targeted to L2L2 behaviors. We showed one 883  
example of how a message sent to students (in the first case) could encourage one student to 884  
share an interesting idea with his peers, and support a moment of consultation that led the 885  
group to change its strategy for solving the problem at stake. The adaptation of the message to 886  
the group's needs originated from the scrutiny of the teacher over the Metafora plan that served 887  
as the footprints of the progression of the group in problem solving. In spite of this success, in 888  
the other courses, the teacher did not use the message tool as she found it difficult to map 889  
openers and messages to the deployment of group work. 890

To sum up, we could list the main practices that emerged during the experiments we 891  
instigated (first research question) and we showed the centrality of Metafora in their enactment 892  
(second research question). An important question remains, though whether these practices are 893  
desirable. 894

## **Are the practices observed desirable according to L2L2 objectives?** 895

The answer to this question can be given by observing the smooth integration of these 896  
practices with the teacher's actions, which are ostensibly directed at promoting L2L2. Indeed, 897  
in both the first and the second case, the teacher incessantly enunciated the envisioned 898  
components of L2L2: collective reflection, mutual engagement, and peer group assessment. 899  
This articulation smoothly combined with inquiry, problem solving, and argumentation activ- 900  
ities. For example, in both cases, the teacher chose an ambiguous picture for presenting a 901  
challenge leading to disagreement among students and to a subsequent LASAD discussion. 902  
The teacher also provided a half-baked plan for boosting a collective plan of action. The 903  
teacher used the Plan map projected by the presenters to contrast between the possible 904  
interpretations and to push students to rely on possible evidence for grounding conflicting 905  
interpretations. She amplified uncertainty and ambiguity (e.g., "is it certain?", "are you sure?") 906  
in order to allow the creation of a student discussion space, instead of telling them what is the 907  
right answer. Here also, this mediation led students to be more aware of their differences of 908  
opinions, and to engage in their resolution. Finally, as the teacher realized that students are not 909  
able to analyse more deeply the stroboscopic picture, she decided to encourage students to turn 910  
to the "Juggler" microworld, which is integrated in Metafora for experiencing ballistic 911  
trajectories and measuring positions of bodies in motion at different times. In other words, 912

the practices mediated by the Metafora system fitted the actions of the teacher who exhibited his/her commitment to L2L2 objectives. The third case showed the unguided emergence of peer group assessment when students argued together and brought snapshots of their inquiries in their argumentative maps. This practice clearly realizes L2L2 objectives.

## Concluding remarks: is L2L2 a clear educational goal and was it attained?

We have presented L2L2 as a coherent educational goal directed at small groups to promote their learning to collaborate when facing L2L tasks – i.e., inquiry and argumentation. The mosaic of practices we just listed looks like a *potpourri* that challenges this coherence. The conjunction of collaboration, inquiry and argumentation seems to convey eclectics rather than coherence. However, the experiments we undertook provide a more integrative picture. For example, the integration between inquiry and argumentation achieved by referable objects that bring critical moments of experience in e-argumentative maps led to *peer group assessment*: In this particular context, peer group assessment is perceived as a need to confront an experience with a microworld as a way to support one's own claims. It is probable that peer group assessment hardly emerged in the other cases because it was not felt as a need.

A more direct sign of the coherence of L2L2 is its context: the task at stake is too difficult for the individual and collaboration is felt as a necessity. *Collective reflection* afforded by the Metafora plan, enables students to report on past moves or to undertake an on-going plan of future moves. The common space leads students to account for past efforts of members of the group and for the co-elaboration of material on which they are reflecting. Mutual engagement is omnipresent in the two first cases: in Ram's decision not to contribute with an insight that would have shown that, again, "he was right", towards a communal effort to solve the problem initiated by Walid and Rose. Mutual engagement was also visible in the second case – in the remodeled plan the group decides to construct after receiving the half-baked plan. It is also salient when Ely does not repeat his statement (that the ball reached the floor) but prefers to put a question mark to indicate uncertainty on whether the player first threw the ball up or just dropped it: the group doubts it, although he has his own opinion. The interaction with his group through the Metafora tools led Ely to take more responsibility towards the group's position; he detached himself from his previous argument to a more general statement ("there is some disagreement here" rather than "I don't agree with..."), which suggest inclusion rather than exclusion. During the further sessions, the group was mature enough for a full-fledged collaborative work including experiencing ballistics with a microworld, collecting data and analysing them, to reach the desired solution. Again, collaboration is seen as the necessary way to handle the complexity of inquiry and argumentation. Students perceived this necessity because of the affordances of Metafora.

Do the three cases show that L2L2 was mastered? The first case alludes to a progression in L2L2: At the beginning, Ram used to dominate his group and even to induce his peers into his mistakes. In the last activity, he gave up a good idea of his own when he saw that it could impair the workflow of his peers and helped them elaborate their ideas. The sacrifice of a good personal idea for the sake of the advancement of the group is a laudable act, but, sometimes, the benefit of the group consists in the best ideas of the individuals. Anyway, the overall functioning of the two groups in the last activity with respect to collaboration in inquiring and arguing was remarkable. They became a 'thinking group' (Stahl 2006) in spite of the high

tension between cognitive and social perspectives that argumentation or inquiry imposes. Both groups in the first case designed an impressive challenge for promoting group learning according to the three envisioned characteristics of group work – including hints for collective reflective or collective responsibility. The fact that peer assessment was rare suggests that the balance between the good functioning of the group as an entity and the best results in inquiry and argumentation was not reached: peer assessment in inquiry should be highly critical concerning the ideas at stake but respectful concerning the relationships in the group. The second case showed the enactment of desirable practices but the support of the teacher was incessant. As for the third case, it presented a nuanced situation, as one of the instances boosted algebraic thinking but the second one impaired it, mainly because an adult did not mediate it.

To conclude, L2L2 seems to be a worthy goal and the experiments we undertook showed that L2L2 was partially promoted. The first cycle of design-based research on L2L2, which we described, leaves many issues still open. Perhaps the major issue is how to motivate students to work in small groups committed to L2L2. The classical goal achievements – mastery and ability goals – does not seem to fit the usual motivation of students. They must somehow be motivated to engage in a learning task, to be accountable to the advancement of the best ideas and to care for sustaining the group.

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