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Orchestrating the emergence of conceptual learning: a case study in a geometry class

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Abstract

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This paper is about orchestrating the *emergence of conceptual learning* in a collaborative setting. We elaborate on the idea of *critical moments* in group learning, events which may lead to a particular development at the epistemic level regarding the shared object. We conjecture that teachers' identification of critical moments may help them guide students to the emergence of conceptual learning. The complexity of small group settings in classrooms prevents teachers from noticing these critical moments, though. Here we present an environment, SAGLET (System for Advancing Group Learning in Educational Technologies), based on the VMT (Virtual Math Teams) environment (Stahl

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2009), which allows teachers to observe multiple groups engaging in problem-solving in geometry. SAGLET capitalizes on machine learning techniques to inform teachers about on-line critical moments by sending them alerts, so that they can then decide whether (and how) to use the alerts in guiding their students. One teacher in an elementary school used SAGLET to help multiple groups of students solve difficult problems in geometry. We observed how the teacher mediated two cohorts of multiple groups at two different times in a mathematics classroom. We show that in both cases the teacher could detect the needs of the groups (partly thanks to the alerts) and could provide adaptive guidance for all the groups.

Introduction: the scarcity of research on orchestration in CSCL environments

The CSCL community has always considered the small-group setting as the natural context for collaborative learning. Initially, the tools developed for facilitating group work did not give the teacher a particular role (Hakkarainen 2010), as it was conjectured that the affordances of these tools would encourage students to engage in learning autonomously and productively (Stahl et al. 2006). However, the CSCL community gradually began to recognize the importance of human guidance. Accordingly, many studies were conducted in laboratory conditions with a small group working in the presence of an instructor. Although the founders of the CSCL movement aimed at educational change, the setting of one small group guided by one teacher did not fit this aim. The proper setting for human guidance of collaboration to produce educational change is the classroom, where many students gather in the same place at the same time. The move from laboratory conditions to the classroom is risky, though, since the multiple constraints of the classroom setting naturally encourage the mere transmission rather than the co-construction of knowledge.

Indeed, the classroom is a highly complex setting for guidance. It requires the teacher to manage real-time decisions in multilayered activities in the presence of multiple constraints (time, discipline, assessment, energy, and space constraints as well as curriculum relevance). Dillenbourg (2013) called this kind of management *orchestration*, and stressed the enormous challenge that teachers face in handling these constraints. The multiple constraints involve not only instructional design, with its traditional components – the material to be taught, the learners' traits and the way knowledge is constructed, but also extrinsic constraints: activities designed to be adaptive, activities designed to be contingent – that is, dependent on what the learners produce – and extraneous events, such as a student dropping out of a group (Dillenbourg 2013). Dillenbourg, who coined the term *orchestration*, is among the founding fathers of the CSCL community. The CSCL community hoped to use technologies to create profound educational changes – e.g., for helping teachers facilitate collaborative processes within groups in classrooms. The term orchestration might have been chosen because, as in the musical realm, where the conductor interacts with sections of the orchestra, the teacher interacts with groups – a more feasible role than interacting with many students individually. The use of CSCL tools for groups adds to this feasibility, since technologies leave traces: teachers can scrutinize students' past actions and reveal their needs. Dillenbourg pointed out that technologies can help in the design of adaptive and emergent activities, but generally cannot help with many other constraints that partly originate from the very use of these technologies, so that great expectations were followed by frustration. Dillenbourg introduced the term orchestration to delineate these constraints: orchestration involves all aspects of teaching management in classroom context, and is more about the coordination of activities (classroom discussions, individual work, small-group collaboration, etc.), than about adapting teaching interventions to individual needs.

Dillenbourg's contribution was not only theoretical but also practical, as he suggested that some of the difficulties involved in orchestration are surmountable. He provided a list of design principles for orchestration, among them *control*, which means that the teacher is in control of what happens in the classroom, and *visibility*. Several tools illustrate these principles. For example, TinkerLamp hardware (Zufferey et al., 2009) enables the teacher to visually scan the whole class and replace digital objects with physical ones. Schwarz and Asterhan (2011) described the Argunaut system, which allows what they called the *e-moderation*[1] of multiple small-group synchronous discussions in parallel (with a CSCL tool) in a school setting. This system helps the teacher control what happens in all the discussions through diverse awareness tools graphically representing the argumentative characteristics of the on-going discussions. Teachers can intervene in light of these aids. Cuendet and colleagues (Cuendet et al. 2015) reported on orchestration in consecutive and diverse activities. They showed how teachers adopted a blended approach to integrate a CSCL tool to help apprentice carpenters learn the skills of their trade. These tools show that orchestration in classrooms is possible – that teachers can handle multiple constraints while teaching a class.

The studies described so far focus more on the conductor than on the orchestra. They do not focus on the music resulting from the interaction between the conductor and the players – emergent learning in interactions among learners mediated by the teacher. For example, Cuendet showed how a CSCL tool is used in an authentic and complex setting (vocational education) in which schoolwork was coordinated with workplace activities, but he did not find any gains in learning. The Argunaut system helped teachers develop sophisticated strategies for boosting the quality of the discussions, by posting messages to groups or to individuals in the groups. E-moderation focused on the role of a teacher in only one kind of activity – argumentative discussions. Schwarz and Asterhan (2011) showed that the moderator's actions had a positive impact on the flow of the discussions: students related more to one another by expanding, challenging, or refuting arguments. However, the productivity of the guidance was not considered beyond these factors.

In this paper, we aim at studying another aspect of orchestration, which Dillenbourg did not stress (although he did not exclude it either). This is the adaptive guidance of groups in the classroom context. One may ask why not reserve the term orchestration to managerial aspects, and to leave more fine-grained aspects of teaching out of the orchestration realm. Our position is that any activity during which the teacher may intervene by interacting with an individual or a group of his choice, while having some control over all students in a classroom context, involves orchestration. The practice we will describe here – the adaptive guidance of multiple groups, is a kind of orchestration. So far, such a practice has been envisaged by many as impracticable. For example, Cobb, Yackel and colleagues described how teachers animate collective discussion and collective reflection in whole-class forums to promote the establishment of socio-mathematical norms for knowledge construction (Cobb et al. 2001; Yackel and

[1] Schwarz and Asterhan used the term 'moderation' to indicate that guidance is caring and at the same time non-intrusive. While Schwarz and Asterhan did not use the term 'orchestration', a posteriori, moderation can be considered as a type of orchestration.

Cobb 1996) and pinpointed the difficulties they encounter in this endeavor. As mentioned by Yackel (2002), the success in this enterprise depends on teachers' connecting their interventions to the progress in the ideas of the whole group, which is an almost impossible task. Cobb and Yackel rarely used the term *orchestration* to describe the teacher's guidance. We conjecture that this omission is not fortuitous. The term 'orchestration' alludes to a great sensitivity of the 'conductor' to the players. Traditional classes in which 30 individuals come to grips with cognitive challenges are not a proper setting for teacher to care for more than a few students at some privileged moments.

In the research we present here, we investigated how teachers can monitor and facilitate group learning when engaging on tasks in parallel. We provide an example of adaptive orchestration, which involves the *emergence of conceptual learning*. The term *emergence* points at a process during which new ideas arise, that may lead to conceptual gains. The orchestration we study takes into consideration special moments that are propitious for this kind of group learning. Our starting point in this paper is the conjecture that when teachers can recognize what we call *critical moments,* orchestration of parallel groups engaged in a task inviting the emergence of conceptual learning is possible.

Key moments and critical moments

Time is a key issue in learning. When we communicate our past experiences, interact on a task, or plan future events, we create spaces where learning can occur. Especially when environments are designed to trigger conceptual learning, there are moments in social interaction – meeting points – which lead to further trajectories of participation (Ludvigsen 2009). This Q7 influence is especially understood to occur at some higher level of structure (Lemke, 2001). Q8 Damşa and Ludvigsen (2016) further developed this idea to define *key moments* in interactions (see also Webster and Mertova 2007), meaning actions or sequences of actions at the epistemic level that trigger subsequent actions and lead to a particular development regarding the shared object. It is not easy to compile a list of such moments. They are recognized by researchers who "analyse the knowledge objects fulfilling interaction-enhancing roles through their active employment in the process over time, by unveiling the relationship between the discursive interaction, the materialization of knowledge through various actions and the temporarily stabilized content of the knowledge object" (p. 8). This recognition occurs after sequences of interaction take place as result of intensive efforts. Damşa and Ludvigsen (2016) did not provide categories of key moments because they are highly dependent on the situation. They nevertheless indicated the epistemic character of interaction, as group members engage in a systematic discussion of concepts at issue, elaborate them, and keep track of the key concepts. Damşa and Ludvigsen's fundamental insight is the importance of the production of *object drafts* for the emergence of key moments.

Damşa and Ludvigsen's idea of key moments inspired us to elaborate what we call *critical moments* – moments in which the teacher's (lack of) intervention may lead to a particular development (positive or negative) at the epistemic level regarding the shared object. For example, if the teacher notices that a group of students is idle, she may decide to intervene (say, by asking the group whether they are having problems, or simply by asking the group to be active). We refer to critical moments in a teaching context, and posit that if the teacher is aware of these moments, she may act on the fly to increase the productivity of the interaction. The notion of productive interaction (Damşa, 2014) involves intellectual interdependence emerging Q9 from (joint) actions directed toward shared goals and an increased understanding of conceptual knowledge. In other words, if teachers are aware of critical moments, they may intervene to improve the students' further trajectories of participation at the epistemic level.

In contrast with key moments that are recognized a posteriori*, critical moments* are potentialities only. God's eye is then necessary to provide them to teachers in a timely fashion, unless the teachers are willing to be satisfied with imperfect information. Research findings can provide such information about moments, which have been found to be correlated with beneficial or detrimental effects. Correlation does not insure that a particular occurrence in social interactions will impinge on the further course of the learning trajectory. However, we conjecture that informing the teacher about this correlation – a potentiality for a beneficial or a detrimental effect – could help her in her interventions. We describe times when we informed teachers that they might be interested in intervening. Romero 2010has synthesized research on time-on-task and student engagement to show that they may be relevant to learning.[2] Monteil (1989) showed the importance of *social validation* of the correctness of solution for further learning gains. D'Mello et al. 2014showed that moments of confusion are often beneficial for learning. Asterhan and Schwarz (2016) demonstrated the beneficial effect of challenges and explanations on conceptual learning. These studies suggest that prolonged moments of idleness, non-validated achievements, off-task engagement, and deliberative argumentation are critical moments in social interaction (to be sure, nothing ensures that specific moments of idleness or of non-validated achievements will necessarily be detrimental to learning). In light of these results, we formulated a list of seven types of occurrence: (a) idleness, (b) off-topic talk, (c) technical problems, (d) explanation or challenge, (e) confusion, (f) correct solutions and (g) incorrect solutions. Our general conjecture is that if teachers are informed about these moments, their intervention may change the development of the interactions to more beneficial (or to less detrimental) at the epistemic level – towards the emergence of conceptual learning.

We give an example to concretize the subtle relations between key moments, critical moments, alerts, and emergent conceptual learning. If the teacher receives an alert that a group of students gave a correct answer to a difficult task designed to foster conceptual learning, the teacher's social validation turns this interaction to a critical moment. If the answer is correctly justified, or if the teachers asks for a justification, and the justification is correct, the critical moment points at the emergence of conceptual learning. This critical moment becomes a posteriori a key moment if the group of students capitalizes on the object at issue in further activities.

The authors of this paper are active in an interdisciplinary project that involves a CSCL tool and data- mining techniques. In pilot studies, we observed students working in groups (one group for each study). The multiple examples collected helped us identify various critical moments in real time. We do not discuss here the algorithms developed for identifying the critical moments. Our starting point in our research is the conjecture (a rewording of the general conjecture described above) that when teachers are informed of critical moments in real time, this information may help them improve their orchestration of the emergence of learning in a classroom setting. This means that the teacher can use the information about

[2] We are aware of the controversy about the relations among time-on-task, engagement and learning. Our approach is practical, as we claim that teachers should know about moments when their students are not engaging in the task at hand, so that they can decide whether or not to intervene.

critical moments to guide several groups in parallel. We developed SAGLET – a system that allows the simultaneous monitoring of multiple groups working on VMT – a CSCL environment, by informing teachers about critical moments through visualizations.

The SAGLET system and its integration with VMT software

SAGLET supports collaborative learning with software used in classrooms. Numerous technologies have been developed to support small-group learning in single, isolated activities. Unfortunately, these technologies are rarely used in schools. When classes are arranged in several small groups working in parallel (in the same classroom or at different sites), the teacher is generally unable to identify critical moments. SAGLET augments on-line learning environments to include technologies with the ability to (1) use educational software to recognize critical moments of emergent learning in groups that are interacting and (2) present salient information to teachers visually. SAGLET provides a set of alerts that the instructor may use to orchestrate multiple groups engaging in parallel on a learning task. Although such a description of critical moments seems a priori useful, great caution is necessary for using alerts in learning-teaching situations. Alerts may function as distractors for teachers who are busy helping specific groups. In the present study, we integrated SAGLET with the Virtual Math Teams (VMT) software.

VMT includes a Geogebra applet shared by all participants and offers them the opportunity to collaborate on geometrical tasks (Stahl 2009). Figure 1 displays an inquiry space in which small groups can share their mathematical explorations and co-construct geometric figures online (part A). When one participant drags or constructs a geometrical shape, all the others can see the changes of the shape. VMT also provides a chat window (part B), in which students can write their ideas, share them with their peers, and coordinate their actions. Students can scroll up and down to return to previous conversations. Figure 2 presents an abridged version of three tasks that we designed for Grade 5 students. They involve the hierarchical inclusion relations of quadrilaterals. We will see later on that understanding this hierarchy requires a conceptual leap (De Villiers 1994; Fujita and Jones 2007).

SAGLET is based on VMT. It allows teachers to observe on-line the work of groups of students engaged in learning tasks with VMT in different rooms and to intervene whenever

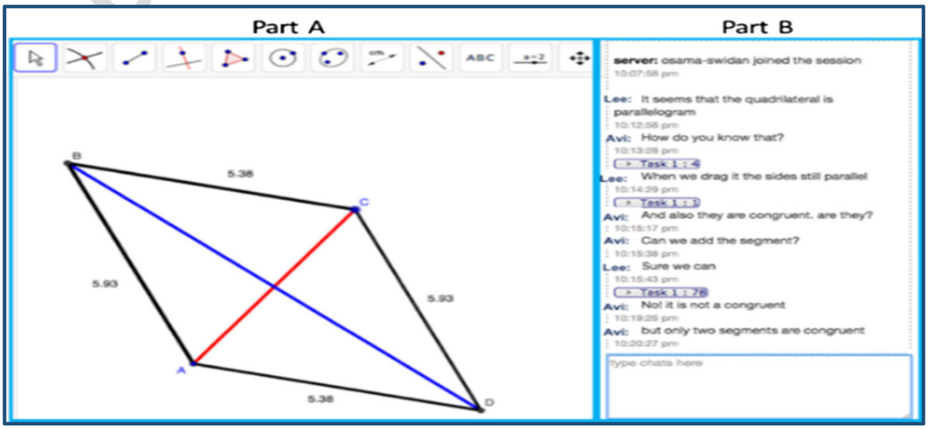


Fig. 1 VMT interface

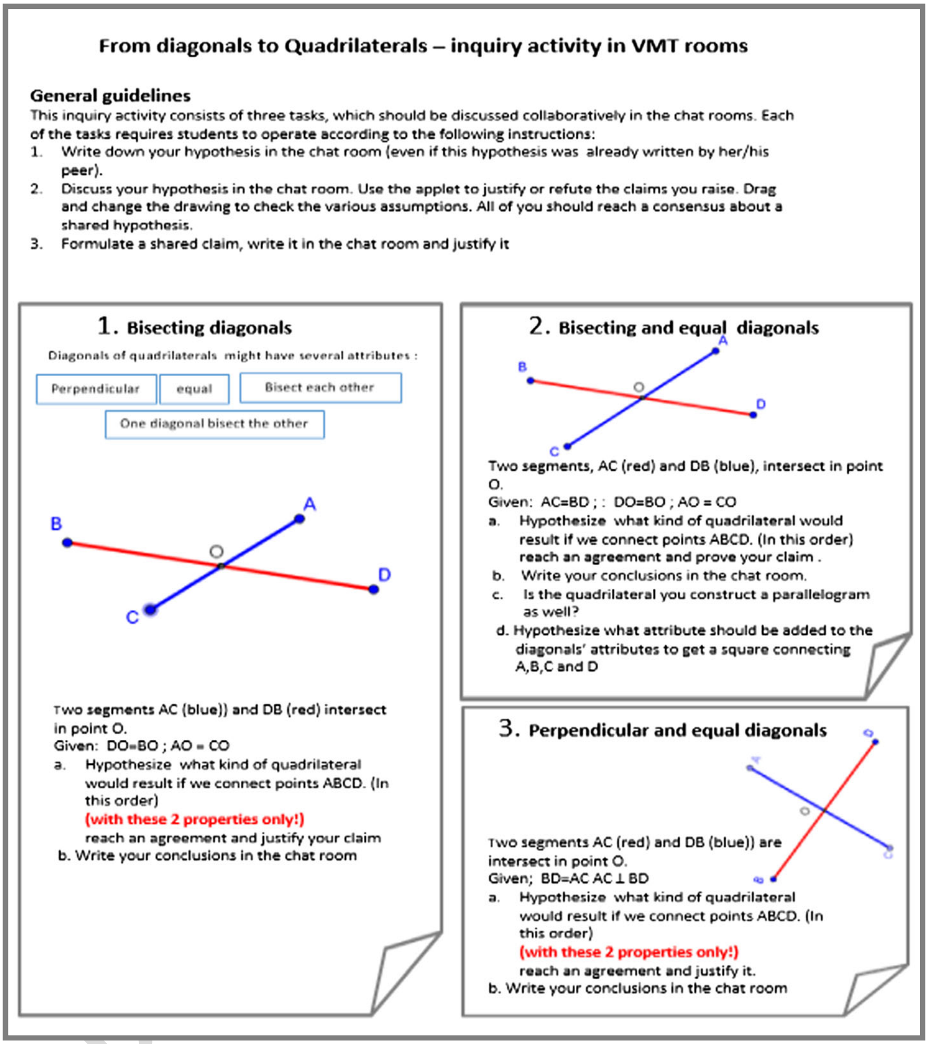


Fig. 2 Three tasks designed for Grade 5 students

they want. As learners progress in their group work, SAGLET informs the teacher of critical moments. Figure 3 shows an example of windows observable by the teacher. In this case, the teacher is informed of a correct solution in room 696 (green frame for the third window from the right) and a technical problem in room 697 (yellow frame for the fourth window from the right). Alerts are easily visible as colored frames. The teacher can disregard them, or may enter the room specified by the alert; she may or may not decide to intervene.

Description of the study

Research question Since the orchestration of conceptual learning of multiple groups solving mathematical problems is a new practice, the most reasonable research step to

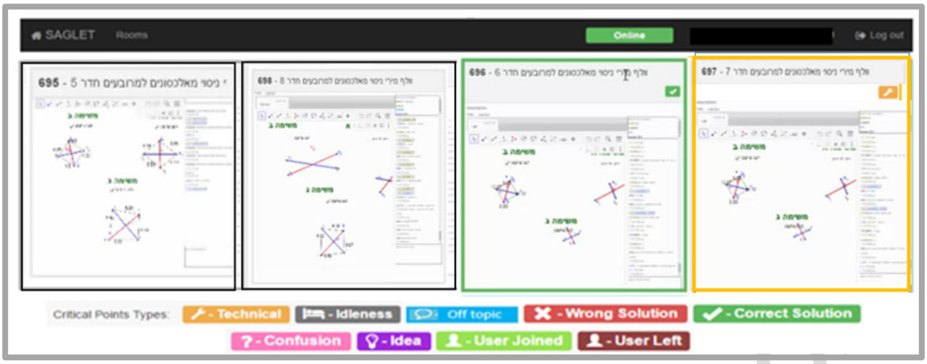


Fig. 3 The teacher is informed of a correct solution in room 696 and a technical problem in room 697

take is to describe its enactment. Our research question involves the tangibility of this practice – whether or not the design actually entails orchestration practices. In other words, our research question is whether a teacher can orchestrate the emergence of conceptual learning in several groups working in parallel with VMT. A positive answer would mean that most of the teacher's actions contribute to the students' learning of geometrical properties, and that these actions are *equitably* distributed among the groups (we use the term *equitably* and not *equally* to stress that the teacher's interventions are sensitive to specific needs, which are obviously not equally distributed). We discuss this issue of equity in the concluding session.

Participants Nineteen students from Grades 5 and 6 participated twice a week in an enrichment program in mathematics during school hours, on a voluntary basis. The two hours were supplementary to the weekly lessons given on a regular basis. The levels of the students in the classroom were mixed, but the students who chose to participate in the enrichment program were all high achievers in mathematics. The two lessons of the enrichment program took place either at the computer lab (when computers were needed), or in the classroom dedicated to high achievers.

Myra, the teacher, has a master's degree in math education. She is an experienced math teacher with 10 years of seniority. She also serves as a pedagogical counselor in pre-service programs for math teachers. She was integrated into the research team of the SAGLET program in November 2016, four months before she served as a teacher orchestrating activities in multiple groups with the SAGLET system.

Procedure The SAGLET system without its alerts was finalized in March 2015. The research team then began analyzing data previously collected from single groups working with VMT. These analyses were used to develop AI-based alerts. The alerts were integrated into the SAGLET system; the second and the third authors of this paper used the system when teaching in small classrooms (with several groups in parallel). Myra joined the research team at that time. She observed the researcher-teachers conducting lessons in geometry with the SAGLET system with up to six groups in parallel. The research team trained Myra in using the system. She then participated in meetings where the research team anticipated difficulties that might be encountered by students in understanding the properties of different kinds of quadrilaterals as geometrical shapes organized in hierarchical relations, and in teaching ways of addressing those difficulties. The pedagogical team then developed (with Myra) a detailed six-week-long teaching unit in which students first familiarize themselves with GeoGebra and the VMT rooms and then study hierarchical relationships in the quadrilateral family. The teaching unit was designed so that students would be introduced first to the properties of geometrical shapes in general, and would then address the more complex issue of the properties of specific shapes. The unit was also designed to develop socio-cultural norms among students – accountability to the other, to reasoning, and to knowledge (Michaels et al. 2008; Schwarz and Baker 2016), and the teacher constantly encouraged the students to collaborate with one another. This was a natural move since, when engaging in the difficult task of finding kinds of quadrilaterals with given properties, students tend to give various erroneous answers (Hershkowitz 1990), and GeoGebra challenges these answers; such a situation requires the coordination of actions. We will not describe the introductory phases of the implementation of the teaching unit at this time. We will only mention that the students participated in two lessons where the teacher orchestrated their work in groups through SAGLET. On the first occasion, the students engaged in an easy task – to identify the properties of given geometrical shapes. At this time, the students became familiar with VMT and SAGLET respectively. Figure 5 displayed later on, shows that at the beginning of the unit, students were sometimes idle and their talk was often off-topic, consequently, alerts were sent to the teacher. The experiment we describe here focuses on the last lesson of the teaching unit, when the students were already familiar with GeoGebra and VMT. During this lesson, the students engaged in the much more challenging task of identifying shapes with given geometrical properties. Importantly, before the last lesson, which is the focus of the present study, Myra and her students had collaborated in a blended setting: Small groups had worked with VMT, communicated with the teacher in a face-to face setting, and had written down their thoughts in the virtual math team rooms. Orchestration with SAGLET followed such activities.

Conceptual learning in elementary geometry

Young students have no major problem identifying the properties of geometrical figures. In contrast, they have great difficulty in finding geometrical shapes that have given properties. For example, several studies have shown that many students have problems with what is called the hierarchical relationships of quadrilaterals (e.g., Fujita and Jones 2007) – that is, the mapping of quadrilaterals onto a set of properties organized in a hierarchical structure. Learning this hierarchical relationship has been shown to help promote the development of geometrical thinking (Fujita and Jones 2007). According to de Villiers (1994), this hierarchical relationship has important functions: (a) It simplifies the deductive systematization and derivation of the properties of more special concepts; (b) It often provides a useful conceptual schema for problem solving; and (c) It sometimes suggests alternative definitions and new propositions. For example, to justify why 'a square is a kite', learners need to be able to inspect its properties. The fact that a square has more properties than a kite should not impinge on the right answer, but in everyday reasoning, it does: children find it difficult to distinguish between critical and non-critical properties (Erez and Yerushalmy 2006). Several researchers have observed that dynamic geometry software like Geogebra offers great potential for conceptually helping many children to see and accept the possibility of hierarchical inclusions; for example, letting them drag the vertices of a dynamic parallelogram to transform it into a rectangle, a rhombus or a square (Jones 2000; Fujita and Jones 2007).

The three tasks in Fig. 2 are about these hierarchical relationships. In Task 1 students are asked which kind of quadrilateral fits the property of having diagonals that bisect each other. The answer is a parallelogram. In Task 2 the two properties of equal and bisecting diagonals fit rectangles (hence, answering that the family of squares is the solution is a wrong answer because it limits the answer to a subfamily). In Task 3 the two properties are that the diagonals are equal and perpendicular. The answer is that many quadrilaterals have these properties but these properties do not characterize any well-known family of quadrilaterals. All these tasks are difficult. They require the third level (of abstraction) according to the van Hiele levels of geometry understanding (Van Hiele 1986). The second and third tasks challenge students' reliance on stereotypes (Hershkowitz 1990): Young students generally mis-identify the fits of rectangles and squares in Task 2, and squares or kites for Task 3. In addition, Task 3 provides an additional challenge – the unexpected answer that there are many unfamiliar kinds of quadrilaterals: it is uncommon in school learning to ask questions whose answers point at an indetermination. Therefore, the tasks provide constraints for the emergence of conceptual learning in group interaction. However, this depends on orchestration.

The experiment

The experiment took place in a computer room. Each student sat at one computer and at the same time was a member of a math team located in a "virtual room" with one or two peers. The experiment was held in two settings. In the first setting, 10 students were arranged in five dyads. Figure 4 displays this setting. In the second setting, nine students

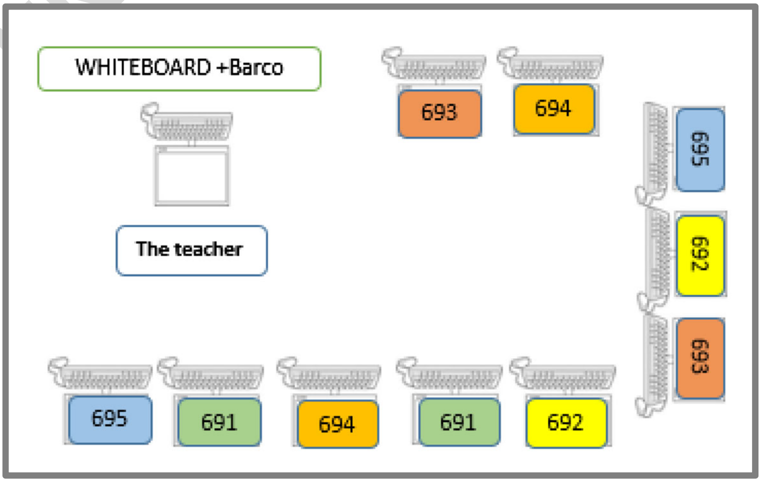


Fig. 4 An arrangement of students in the computer lab

were arranged in three triads. None of the two settings constitutes a full classroom setting. We deliberatively divided the class in two cohorts – one of dyads, and the second of triads, to check whether the teacher could handle the complexity of orchestration in half a class. Importantly, the members of each room were sitting apart from each other. The teacher asked them not to talk. Rather, they were invited to write their claims in the chat window of the VMT system. The teacher explained to the students that if they did so, she could look at whatever they said and would be able to help them when needed. Therefore, although the request not to talk seems a bit artificial, its justification was primarily rooted in pedagogy. In fact, the students accepted this kind of contract with the teacher and remained silent during the whole lesson. The teacher distributed a booklet to each student. The booklet included the three tasks (see Fig. 2) as well as a collaborative script – specific instructions to collaborate with peers and a requirement to reach consensus about the solution (see the abridged version of these instructions in the general guidelines in Fig. 2). Other instructions encouraged students to argue with each other, to justify their claims, and to try to refute the claims of their peers when they disagreed with them. After getting the booklet, the students in each setting were asked to solve the three tasks in a 90-min session.

Methodology

As mentioned, the adaptive guidance of several groups collaborating in parallel has rarely been reported. The natural methodological step for observing this new practice is to adopt the ethnographic approach (Merriam 1998). Specifically, we adopted the Case Study Method. The applicative power of case studies fits the study of guidance of multiple groups in parallel, partly because it cannot be compared to other already known procedures (Eisenhardt 1989; Eisenhardt and Graebner 2007). Moreover, the inquiry is empirical and focuses on a procedure that takes in its natural context –the classroom (Yin 1994, p. 13) while resulting in a thick description, which articulates a full procedure (MacDonald and Walker 1977). Since the group work and its guidance took place in a technological environment, replaying them was quite easy. We chose to apply the technique of *cued retrospective reporting* (Van Gog et al. 2009) to the study of the guidance of several groups in parallel. Accordingly, the teacher could look at all her actions, including how she navigated through the system as well as her mouse and keyboard actions. Two weeks after the experiment, the research team asked the teacher to comment on her actions. She was asked to interpret her behavior and evaluate whether her interventions were productive. Q12

Data collection

Data were collected on three occasions. First, during the experiment, the complete activities of the teacher and the students were video-recorded with the CAMTASIA package, and a video camera captured the entire classroom. Immediately after the experiment, we interviewed the teacher. We asked general questions regarding the teacher's ability to conduct the geometrical lesson with the SAGLET interface. In parallel, we uploaded the logs of the students' chats from the VMT software. Two weeks after the experiment, we organized a cued retrospective reporting session, in which the teacher watched the video that replayed how she navigated between rooms and intervened during the experiment. The teacher was asked to provide interpretations of her interventions during the lesson in the cued retrospective session. This retrospective was recorded, too, and then transcribed. The student work and the cued retrospective reporting session were conducted in Hebrew and the transcription was translated into English by the researchers in this report.

Analysis of the data

Since we were scrutinizing orchestration, we focused mainly on the teacher's interventions. Since the teacher was intervening in previous group work, we described the minimal interactive episodes, which could give the interventions meaning. The teacher watched the video of her work with Camtasia, and commented on her actions in the video (cued retrospective reporting). The comments were then transcribed and arranged in tables in parallel with the VMT protocols and moves. The teacher analyzed these tables, identified episodes in which she intervened, and classified these episodes into categories. The second and third authors of this article checked and refined the categories the teacher identified; they often watched the Camtasia video to validate or refine a category. This procedure produced a list of five *types of interventions*: (a) encouraging collaboration among group members; (b) monitoring and supervising the execution of the task; (c) asking for justifications; (d) scaffolding argumentation; and (e) social validation. We present episodes demonstrating instances of these types of intervention. The types of interventions were validated by the interpretations given by the teacher in her cued retrospective reporting.

Results

Before we delve into the details of types of intervention, we first sketch a global picture of the general flow of the session (both with the five dyads and with the three triads). As shown in Table 1, seven of the eight groups completed at least two of the three tasks and only one group only did not succeed in finishing even the second task. Also, two groups finished the three tasks before the end of the 90 min and another three were still working when the 90 min were over. As we will see, the teacher did not allow students to tackle new tasks before she checked that they had given the right answer with a satisfactory explanation.

These global facts seem to show that the teacher's guidance was an instance of successful orchestration, as she could handle the progression of all the groups toward the successful solution of challenging problems in geometry at different rates. This global appreciation is not sufficient, though, and a fine-grained analysis of the interventions is necessary for describing the nature of the guidance. We present episodes demonstrating instances of the five types of intervention mentioned above.

Table 1 Global achievements of the eight groups

Room Task	691	692	693	694	695	696	697	698
Task 1	V	v	v	V	V	V	V	V
Task 2	V	V	V	V	V	V	Wrong	V
Task 3	Not finished	Not finished	V	–	–	V	–	Not finished

(a) Encouraging collaboration between group members

The first type of intervention consists of prompts for encouraging collaborative behaviors. No algorithm and consequently no alert is provided by SAGLET. However, the teacher quite easily detects moments when collaboration is needed, as in the following example:

Room No	Time	VMT Protocols and moves	Cued retrospective reporting
696	11:07:33	Jo: Is it a parallelogram?	I read the responses of the students and I noticed the disagreement about the right answer. So I decided to ask them to convince each other about the right answer.
	11:07:58	Jo: One moment.	
	11:08:04	Raphael: It's a parallelogram.	
	11:08:08	Jo: it's a square.	
	11:08:20	Raphael: I think it's a parallelogram.	
	11:08:38	Shimmy: I think it's a rhombus.	
No alert	11:08:43	Teacher: You need to reach an agreement about the right solution, all of you!!	

In this short excerpt, we see that the teacher read the interactions between three students and noticed that each of them proposed a different answer. She prompted them to reach an agreement – a practice she had already declared as desirable in group work at the beginning of the lesson. Other instances of this kind of intervention were "an invitation to collaborate" or "an invitation to relate to each other". It is interesting that these kinds of interventions became less frequent in the course of the lesson. Examining the logs of the students' interactions indicates that these kinds of interventions were needed less. This impression is confirmed by the teacher's acknowledgment in her interview that she felt that" encouraging collaboration in group work became progressively unnecessary."

(b) Monitoring and supervising the execution of a task

Another type of intervention that the teacher could undertake is supervising the execution of a task, as in the following example, in which a group of students began solving Tasks 2 and 3 before they completed Task 1:

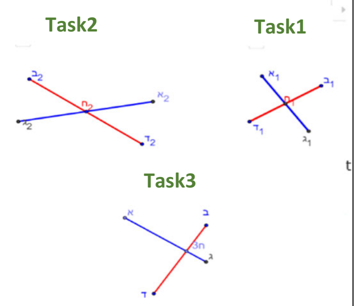
Room	Time	VMT Protocols and moves	Cued retrospective reporting
696	11:01:01- 11:02:44		You can see in the GeoGebra applet that one of students dragged the diagonals of the next task, task 2, and therefore I asked them to concentrate on task 1.
No alert	11:02:45	Teacher: Please concentrate only on the first task! Stop dragging the diagonals of tasks 2 and 3.	

Fig. 5 Alerts informing the teacher of off-topic talk and idleness lead her to enter a room

In this case, even without an alert, the teacher could identify a moment of the non-execution of a task through movements in GeoGebra that are visible even in the miniatures that SAGLET provides (as in Fig. 3). Other forms of this type of intervention appeared. We present one form that did not appear in the last lesson of the unit. In the first lessons, the discourse was sometimes off-topic, or the students were idle for a while. Figure 5 describes such moments when, following an alert about idleness or off-topic talk, the teacher chose to enter the room referred to in the alert, and intervened:

(c) Asking for justifications and (e) social validation

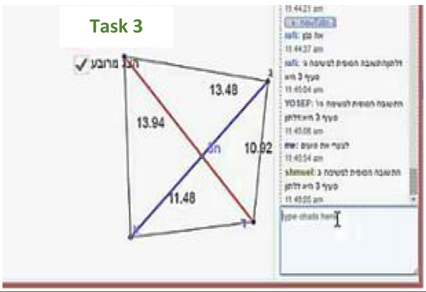
We present two types of intervention that the teacher very often implemented at the same time – asking for justifications and social validation. Here is one example:

Room No	Time	VMT Protocols and moves	Cued retrospective
696	11:38:23	Raphael: Our final answer to task 2 is a rectangle.	I got an alert about a correct answer, so I asked the students to articulate their reasoning.
Green Alert about a correct answer	11:38:47 11:38:50	 Teacher: Excellent! Teacher: Please justify!	

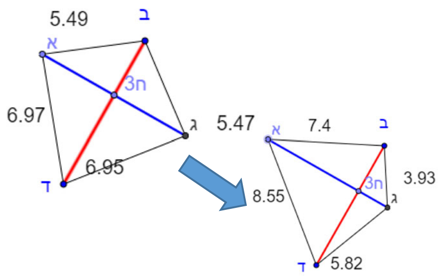
As in the last example, a clear declaration of one of the members of a group, Raphael, that he had reached the answer led to an alert. Since the answer is correct, the alert is green. This kind of alert invites intervention of the teacher with almost the same degree of urgency. In the present example, the students decided that they arrived to their final answer. The teacher's intervention is then not an interruption. It is urgent, though. The teacher undertakes two distinct actions. First, her "Excellent" serves as social validation. She tells the members of the group that they worked well. As mentioned, this is a very important action. Second, she asks the students to justify their answer, to elaborate their claim into an argument. We put these two actions together because they always occurred at the same time.

(d) Scaffolding argumentation

So far, all the interventions of the teacher that we presented fit a quite conventional role – that of a kind of police officer who checks that the students are doing what they are supposed to be doing. Even the request to give justifications is peremptory, and students know they have to provide the expected justification. The interventions are necessary and thus it is impressive that, with the help of SAGLET, the teacher can check that her students are active and on the right task. However, the teacher's role in helping small groups is additionally envisioned in a more subtle way – as a facilitator of knowledge construction. We will see now that this is exactly what Myra did in the difficult context of several groups in parallel. The type of intervention we will illustrate now is called "scaffolding argumentation". For this kind of intervention, the teacher is faithful to her mission to help the students without giving decisive answers right away. For example, she may suggest actions that lead students to refute their own arguments. The following moment exemplifies this. It begins with an alert provided by the system, stating that a wrong solution has been proposed (our NPL-based algorithms identify sentences such as "our final answer to…is…):

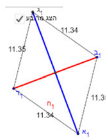
Room No	Time	VMT Protocols and moves	Cued retrospective
696	11:44:37 11:45:04 11:45:06	Raphael: Ha…you're right. Raphael: Our final answer to task 3 is a kite Jo: Our final answer to task 3 is a kite 	I got an alert about wrong solution so I read and gave them a hint to check whether the properties of a kite exist in the quadrilateral they dragged
Red Alert about a wrong answer	11:45:54	Teacher: I'm sorry but your answer is wrong! Does the quadrilateral you see have the properties of a kite?	
The right answer	11:46:43	Jo: So it is just a 'no name' quadrilateral?	

The students are solving Task 3 – finding quadrilaterals whose diagonals are equal and perpendicular to each other. We see here that the teacher's attention was drawn by an alert that the students had declared that they found the solution, but the alert suggested that this solution was wrong. Such a declaration requires an immediate intervention. The teacher used the GeoGebra shapes (with equal and perpendicular diagonals) that the students had generated to challenge them. Her question "Does the quadrilateral you see have the properties of a kite?" is in fact a challenge. Jo then suggests that the answer is a 'no name' quadrilateral. It appears that the cognitive conflict the teacher provided by pointing at a random quadrilateral leads Jo to recognize that the two properties can fit no-name quadrilaterals. We present another example of the scaffolding of argumentation, in which the teacher intervenes directly through the GeoGebra applet:

Room No	Time	VMT Protocols and moves	Cued retrospective
696	11:52:14 11:52:28 11:52:37 11:53:37	Jo: It should be a square. Raphael: NO! It shouldn't. Jo: Why not? Sam: The diagonals are perpendicular and congruent as well and the only quadrilateral that has these attributes is a square.	I saw that the students argue but could not reach consensus about the correct answer, so I ask for the control of the applet and introduced them counter examples to their claims
Using the chat window No alert	11:53:50 11:54:16 11:54:27 11:54:32 11:54:37 11:54:47 11:56:01 11:56:14	Teacher: Please, give me control over GeoGebra  Teacher: Is it a square now? Jo: No! Raphael: No. Teacher: So what do you see? Jo: A kite Raphael: Just a quadrilateral with no special name Teacher: That's great, explain why. Raphael: Because this quadrilateral does not have any special property.	

We see that Jo and Raphael do not agree about the result of Task 3 – finding quadrilaterals with diagonals that are equal in length and perpendicular to each other. The teacher does not receive any alert here but identifies this disagreement. Jo's affirmation that the answer is a square fits the classical reasoning based on stereotypes mentioned above. The teacher acknowledges that she is intervening because of the disagreement, and this time directly intervening in the GeoGebra space. She drags the given shape, and the dragging preserves the two given properties – orthogonality and equal length of diagonals. The three students observe her dragging, which results in a 'no-name' quadrilateral. Jo, who sees that the properties are preserved, agrees that the shape is not a square but claims that the shape is a kite. After observing the teacher's dragging, Raphael can explain that the quadrilateral does not necessarily have any special property. This example may present the teacher as an intruder in the collaborative work of a group: two students did not agree on the solution and the teacher could have left the students settle their disagreement on their own. We have looked at the whole interaction of the three children, and this particular intervention seems reasonable, although perhaps not necessary. In general, the teacher's interventions and their timing, which constituted her orchestration, were reasonable.

We present a last example of scaffolding argumentation, where the teacher intervenes through the chat section, rather than through the GeoGebra applet:

Room No		Time	VMT Protocols and moves	Cued retrospective reporting
697		11:13:56 11:13:58 11:14:18 11:14:49 11:15:26	Yotam: Let's first try to get the sides equal. Abe: Give me control over GeoGebra David: Why do you need to get the sides equal? Abe: I would like to try something… David: Look at the information about rhombuses.	
Using the chat window No alert		11:15:45	 Teacher: Are the properties of the rhombus preserved after dragging the shape? Or might it be another kind of quadrilateral?	Members of this group have not yet really understood the task and I felt like I needed to guide and direct them more than other groups, helping them to understand.

In this case, students are engaged in Task 1 and try to drag one of the vertices of the given quadrilateral to turn it to a rhombus. Without being given any alert, the teacher notices that the students do not understand the task: They should not be trying to find particular shapes that have the given property (bisecting diagonals) but should inquire about the general shapes that have these properties. The teacher asks "Are the properties of the rhombus preserved after dragging the shape? Or might it be another kind of quadrilateral?", which is a challenge to their claim. We present a last example, which integrates three types of interventions: the request for justifications, social validation and supervision of the task:

A combination of request for justifications, social validation, and supervision of the task.

Room No	Time	VMT Protocols and moves	Cued retrospective
698	11:20:02	Teacher: Please justify why this is a parallelogram.	I entered the room when I saw a green alert about a correct solution. Then I asked the group for justifications for their answer and when I read the correct justification they gave, I complimented them, since I wanted them to feel good about their learning.
	11:20:45	Jill: Because it has two pairs of opposite equal sides.	
	11:20:54	Teacher: Excellent!	
Green Alert about a right answer	11:21:02	Yirme: Because the opposite sides are parallel.	
	11:21:06	Teacher: You may start the next task. 	

The students are solving Task 1. They find the right answer – the family of parallelograms. After a green alert, the teacher enters the room and asks the students to justify their answer. Jill and Yirme use two different *critical properties* of parallelograms. The teacher is fully aware of the importance of social validation for learning processes; as she declares in the cued retrospective report, "I complimented them, since I wanted them to feel good about their learning". The teacher, who could monitor the execution of the task, suggested that the students go on to the next task. This episode exemplifies a typical orchestration move, which was also common with other groups. This episode uncovers a somewhat problematic aspect of the teacher's guidance. The students are fifth graders and are not expected to provide mathematical proofs for their claims based on congruence theorems. However, the use of Digital Geometry tools makes it possible to show that general shapes have distinct properties by dragging a randomly generated shape whose properties have been defined in advance. The teacher does not use this feature of GeoGebra here. When the students give the right answer, probably drawn from observation, she compliments them. The students might have benefited from a request like "Show me that you are right" when they state, "Because it has two pairs of opposite equal sides".

We have presented five types of interventions. For some of the examples, no alert was given before the teacher intervened. During the lesson, the teacher went from one room to another 47 times. During the first three minutes, she did so four times and, as she stated, "At the beginning, I went from one room to the other to see how the groups were approaching the problem without going deeply into their work; I only wanted to check that they understood the task". At the fourth minute, the teacher saw an alert. Of the 43 times that she went from one room to another, 21 were triggered by an alert. Figure 6 displays the repartition of these interventions during the first 32 min of the lesson with the three triads of students. Figure 7 shows the scattering of the interventions across all the groups. This scattering is not uniform. For example, in room 698, the teacher intervened 14 times between 9:10 and 9:21, and the

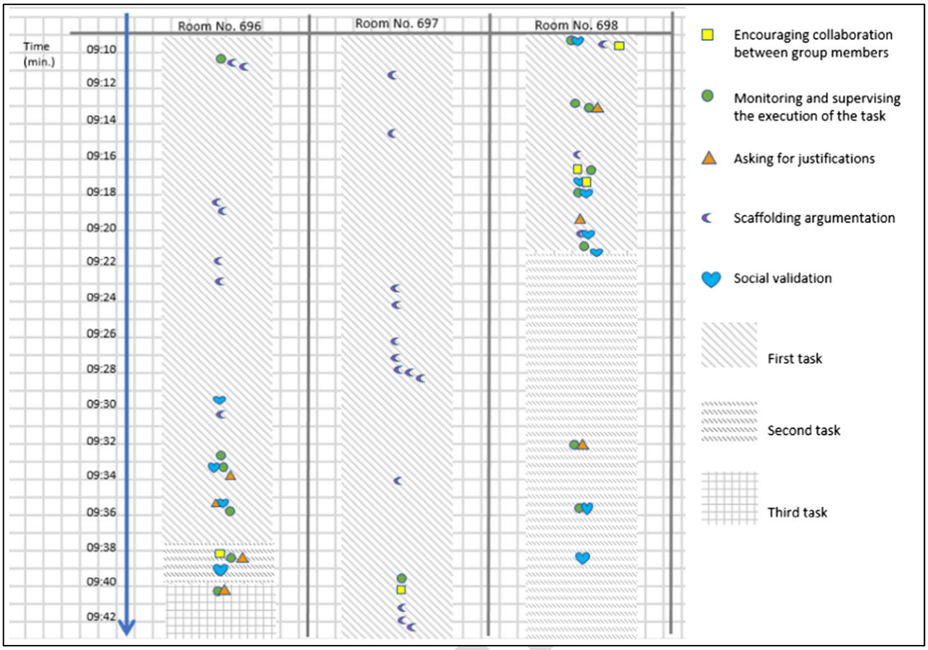


Fig. 6 The scattering of the teacher's interventions during 32 min

interventions were of all types. The group in this room finished Task 1 and moved on to Task 2. At the same time, the teacher intervened in rooms 696 and 697 as well. In room 696, the teacher intervened nine times between 9:22 and 9:37, resulting in the successful completion of Task 1 and the authorization to go on to Task 2. At the same time, in room 697, the teacher scaffolded argumentations eight times. However, in spite of her efforts, the students did not succeed in solving the task. The students in room 698 worked without guidance, apparently because the teacher progressively realized that this group did not need assistance. We also see that in room 696, the group swiftly solves Task 2 and goes on to Task 3. We also see that most of the time the different groups are working on different tasks, and that the teacher swings between these groups. It is remarkable that the teacher was able to intervene in the three groups between 9:38 and 9:40, as these groups were working on different tasks!

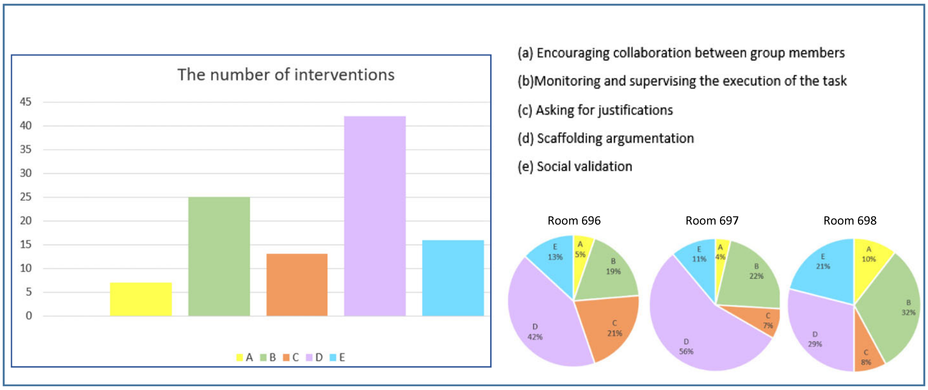


Fig. 7 The repartition of the types of interventions in the cohort of three groups of three students

Figure 7 displays the distribution of the types of teacher's intervention in the same three triads of students. We see that the scaffolding of argumentation is the most frequent type of intervention. As mentioned, this kind of intervention fits the CSCL spirit, where guidance is ancillary to the co-construction of meaning. This intervention presents orchestration as an activity that involves meaning-making. We saw that the forms of this scaffolding were diverse, including challenges or refutations expressed either in a chat mode or with GeoGebra. The next most frequent intervention was monitoring and supervising the execution of the task – an intervention that treats orchestration as a way to control group work in the classroom without delving into meaning-making. In contrast, the request for justifications and social validation treat orchestration as an activity that supports meaning-making. The least frequent type of intervention was the encouragement of collaboration. After one preparatory session with SAGLET, the students were already accustomed to collaborate, which indicates that VMT *affords* collaboration.

Discussion

The research question we investigated in this study was whether a teacher can orchestrate the emergence of conceptual learning in several groups working in parallel with VMT. SAGLET provided *control* over all groups, as the teacher could scrutinize the (miniature) rooms of several groups in parallel. Also, alerts about critical moments were *visible* through the colored frames of the representations of the rooms when such moments occurred. We mentioned that all eight groups solved the first task correctly and seven did so for the second task. This is an outstanding achievement, as the tasks require the students to match properties to geometrical shapes correctly. This is a difficult step, which matches the third level (of abstraction) of geometry understanding according to the van Hiele hierarchy (Van Hiele 1986). These facts indicate that the teacher orchestrated the emergence of conceptual learning in a geometry class. SAGLET allowed the teacher to handle different paces of progression that led her to combine the observation of and intervention in groups' work on different tasks. We identified five kinds of intervention. Figures 6 and 7 show the great diversity in the distribution of these interventions among the groups. This diversity does not prove that the teacher's guidance was always adaptive in the sense that it accounted for the needs of the students. However, the examples we presented suggest that this is generally the case. Figure 6 clearly stresses the managerial aspects of the teacher's generally adaptive guidance, and indicates that this adaptive guidance was an orchestration of multiple groups working in parallel.

Although we concentrated on the interventions of the teacher to state that we observed orchestration of the emergence of conceptual learning, the contingency of the teacher's interventions with the students' actions suggests that this orchestration happened at two layers, the second being at the layer of emergent learning. It is also remarkable that the multiple constraints that characterize classroom activity (discipline, energy, etc.) were handled in the lesson: Young students remained silent and communicated through SAGLET only to solve difficult problem during a short period. The design of VMT and of SAGLET, that enables group inquiry (with Geogebra) and discussion simultaneously, opens the door for classroom activities in which orchestration of the emergence of conceptual learning is possible. This is an important finding, at the time research efforts on orchestration based on Learning Analytics still focus on the identification of design principles (Van Leeuwen and Rummel 2017).

Our work has insight for the field of learning analytics that focuses on the design of educational technologies that are informed by analysis of students' interactions (Baker and Inventado 2014). The accessibility of log data of student group interactions from SAGLET, can be coupled with intelligent machine learning algorithms for automatically detecting critical moments, thus improving the teacher's ability to monitor and support group learning in the classroom (Segal et al. 2017).

We are aware, though, that the teacher worked twice with half a class. Providing ten rooms of triads in SAGLET is a challenge that the teacher did not experience yet. In spite of this limitation, the combination between managerial and adaptive aspects confers to orchestration a new role – in the same manner as a conductor, who is committed to the global execution of musical creations, but at the same time interacts with certain players in the development of their interpretations.

We preceded the term *conceptual learning* by the term *emergence* in this paper because we concentrated on one lesson without checking further trajectories of participation among groups of students. In her interventions, the teacher never gave the right answer but encouraged collaboration and supervised the execution of the task; when she noticed that knowledge construction was at issue, she asked for justifications, scaffolded argumentation, or validated a conclusion. These moments were critical moments that fostered emergent learning. In summary, although solving the three tasks correspond to a high-level of reasoning in Geometry, our focus on one lesson does not allow us to refer to *conceptual learning*.

At the beginning of this paper, we based our approach to the facilitation of orchestration on the provision of critical moments of different types. However, in some of the examples we presented, no alert was provided, and this did not prevent successful moments of guidance. The teacher could detect the absence of collaboration, or the absence of progression, by simply looking at the miniature representations of group work or directly entering rooms. We are pleased by this state of things. The teacher's actions often originated from direct observation of the groups' progression, and SAGLET enabled the teacher to survey the rooms quite easily. The provision of alerts did not direct the teacher's guidance, but helped her when she felt it could be productive. SAGLET as a 'non-intelligent' system made it possible to reveal a panoramic picture of groups working in parallel. This is an important finding; it shows that computer design turns the facilitation of guidance of several groups in parallel into a feasible enterprise.

In most of the examples we presented where the teacher's intervention followed an alert, the alert was only about right or wrong answers. We mentioned alerts about idleness and off-topic discourse that occurred only during the preparatory session. We could have illustrated alerts about technical difficulties. In other experiments we conducted, the teachers knew how to use these kinds of alert in their orchestration. In the highly motivating context of well-designed tasks in the presence of a research team, students were not likely to be idle or have off-topic conversations. When we presented a list of critical moments at the beginning of this paper, we mentioned alerts acknowledging moments of confusion, or of explanation and challenge. We did not document these kinds of alert in this paper because the reliability of the algorithms generated to produce them is not yet high enough to provide useful information for teachers. We mentioned these critical moments because we believe that it is possible to identify them automatically, and we are on the way to producing this result. Our optimism concurs with considerations of specialists on orchestration in Learning Analytics (Wise and Vytasek 2017). We are not sure, however, that providing a profusion of alerts to teachers helps them in their guidance. The alerts may distract them from their complicated task of orchestrating emergent learning. As shown in Fig. 7, teachers could frequently scaffold argumentation without being provided with any alert. Providing teachers with sophisticated tools may help them or may hinder the process of orchestration, and finding the right balance in this new direction in orchestration is an open and exciting question (see also Van Leeuwen 2015).

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